Алгебра. Самостоятельная работа.

Номер варианта — это номер студента в алфавитном списке курса.

Задача 1. Найдите оба полярные разложения для данной матрицы. Докажите, что найденные сомножители унитарные (соответственно, самосопряженные и положительные).

1) $\begin{bmatrix} 3+4\,i & -2 \\ -4+2\,i & -4-5\,i \end{bmatrix}$

2) $\begin{bmatrix} 2+3\,i & -5+4\,i \\ -2+i & -2\,i \end{bmatrix}$

3) $\begin{bmatrix} 1+4\,i & -6+2\,i \\ -1 & 2-i \end{bmatrix}$

4) $\begin{bmatrix} -1+4\,i & -6-i \\ -1 & 2-2\,i \end{bmatrix}$

5) $\begin{bmatrix} 1+2\,i & -2+3\,i \\ -3-2\,i & 4-4\,i \end{bmatrix}$

6) $\begin{bmatrix} i & -1+2\,i \\ -4-i & 2-6\,i \end{bmatrix}$

7) $\begin{bmatrix} -1+2\,i & -2 \\ -3-2\,i & 4-5\,i \end{bmatrix}$

8) $\begin{bmatrix} -2 & -1+2\,i \\ -4+5\,i & 3+2\,i \end{bmatrix}$

9) $\begin{bmatrix} -1+2\,i & i \\ -2+6\,i & 4+i \end{bmatrix}$

10) $\begin{bmatrix} -1 & -2+i \\ 1+4\,i & 6-2\,i \end{bmatrix}$

11) $\begin{bmatrix} -4+4\,i & 2+3\,i \\ -3+2\,i & 2+i \end{bmatrix}$

12) $\begin{bmatrix} -6+2\,i & 1+4\,i \\ -2+i & 1 \end{bmatrix}$

13) $\begin{bmatrix} -5+4\,i & 2+3\,i \\ 2\,i & 2-i \end{bmatrix}$

14) $\begin{bmatrix} 2+3\,i & -5+4\,i \\ -2+i & -2\,i \end{bmatrix}$

15) $\begin{bmatrix} 1+4\,i & -6+2\,i \\ -1 & 2-i \end{bmatrix}$

16) $\begin{bmatrix} -1+4\,i & -6-i \\ -1 & 2-2\,i \end{bmatrix}$

17) $\begin{bmatrix} 1+2\,i & -2+3\,i \\ -3-2\,i & 4-4\,i \end{bmatrix}$

18) $\begin{bmatrix} i & -1+2\,i \\ -4-i & 2-6\,i \end{bmatrix}$

19) $\begin{bmatrix} -1+2\,i & -2 \\ -3-2\,i & 4-5\,i \end{bmatrix}$

20) $\begin{bmatrix} -2 & -1+2\,i \\ -4+5\,i & 3+2\,i \end{bmatrix}$

21) $\begin{bmatrix} -1+2\,i & i \\ -2+6\,i & 4+i \end{bmatrix}$

22) $\begin{bmatrix} -1 & -2+i \\ 1+4\,i & 6-2\,i \end{bmatrix}$

23) $\begin{bmatrix} -4+4\,i & 2+3\,i \\ -3+2\,i & 2+i \end{bmatrix}$

24) $\begin{bmatrix} -6+2\,i & 1+4\,i \\ -2+i & 1 \end{bmatrix}$

25) $\begin{bmatrix} -5+4\,i & 2+3\,i \\ 2\,i & 2-i \end{bmatrix}$

26) $\begin{bmatrix} -2-4\,i & 5-4\,i \\ 4-3\,i & 2\,i \end{bmatrix}$

27) $\begin{bmatrix} -1-2\,i & 2 \\ 3-2\,i & 4+5\,i \end{bmatrix}$

28) $\begin{bmatrix} -i & 1+2\,i \\ 4-i & 2+6\,i \end{bmatrix}$

29) $\begin{bmatrix} -i & 2+2\,i \\ 4+i & -1+6\,i \end{bmatrix}$

30) $\begin{bmatrix} 2-3\,i & 4+4\,i \\ 2-i & 3+2\,i \end{bmatrix}$

31) $\begin{bmatrix} 1-4\,i & 6+2\,i \\ 1 & 2+i \end{bmatrix}$

32) $\begin{bmatrix} 2-3\,i & 5+4\,i \\ 2+i & 2\,i \end{bmatrix}$

33) $\begin{bmatrix} -5-4\,i & -2+3\,i \\ 2\,i & 2+i \end{bmatrix}$

34) $\begin{bmatrix} -6-2\,i & -1+4\,i \\ 2+i & 1 \end{bmatrix}$

35) $\begin{bmatrix} -6+i & 1+4\,i \\ 2+2\,i & 1 \end{bmatrix}$

36) $\begin{bmatrix} -2-3\,i & -1+2\,i \\ 4+4\,i & 3-2\,i \end{bmatrix}$

37) $\begin{bmatrix} -1-2\,i & i \\ 2+6\,i & 4-i \end{bmatrix}$

38) $\begin{bmatrix} -2 & 1+2\,i \\ 4+5\,i & 3-2\,i \end{bmatrix}$

39) $\begin{bmatrix} -1-2\,i & 2 \\ 3-2\,i & 4+5\,i \end{bmatrix}$

40) $\begin{bmatrix} -i & 1+2\,i \\ 4-i & 2+6\,i \end{bmatrix}$

41) $\begin{bmatrix} -i & 2+2\,i \\ 4+i & -1+6\,i \end{bmatrix}$

42) $\begin{bmatrix} 2-3\,i & 4+4\,i \\ 2-i & 3+2\,i \end{bmatrix}$

43) $\begin{bmatrix} 1-4\,i & 6+2\,i \\ 1 & 2+i \end{bmatrix}$

44) $\begin{bmatrix} 2-3\,i & 5+4\,i \\ 2+i & 2\,i \end{bmatrix}$

45) $\begin{bmatrix} -5-4\,i & -2+3\,i \\ 2\,i & 2+i \end{bmatrix}$

46) $\begin{bmatrix} -6-2\,i & -1+4\,i \\ 2+i & 1 \end{bmatrix}$

47) $\begin{bmatrix} -6+i & 1+4\,i \\ 2+2\,i & 1 \end{bmatrix}$

48) $\begin{bmatrix} -2-3\,i & -1+2\,i \\ 4+4\,i & 3-2\,i \end{bmatrix}$

49) $\begin{bmatrix} -1-2\,i & i \\ 2+6\,i & 4-i \end{bmatrix}$

Задача 2. Найдите оба полярные разложения для данной матрицы. Докажите, что найденные сомножители унитарные (соответственно, самосопряженные и положительные).

1) $\begin{bmatrix} -2/3 - 4/3\,i & -2/3 \\ -5/2 + 1/6\,i & -\frac{17}{6} - 1/6\,i \end{bmatrix}$

2) $\begin{bmatrix} 2/3 - 4/3\,i & -2/3\,i \\ 1/6 - 5/2\,i & -\frac{17}{6} + 1/6\,i \end{bmatrix}$

3) $\begin{bmatrix} -1/6 - 5/2\,i & -1/6 + \frac{17}{6}\,i \\ -4/3 + 2/3\,i & -2/3\,i \end{bmatrix}$

4) $\begin{bmatrix} -1/6 - \frac{17}{6}\,i & 5/2 + 1/6\,i \\ -2/3 & -4/3 - 2/3\,i \end{bmatrix}$

5) $\begin{bmatrix} -\frac{11}{6} - 7/6\,i & -5/6 - 3/2\,i \\ 1 - 5/3\,i & 1/3 - 7/3\,i \end{bmatrix}$

6) $\begin{bmatrix} 5/6 - 7/6\,i & -3/2 - \frac{11}{6}\,i \\ 7/3 + i & 1/3 - 5/3\,i \end{bmatrix}$

7) $\begin{bmatrix} -7/3 + i & 5/3 - 1/3\,i \\ -7/6 + 5/6\,i & 3/2 - \frac{11}{6}\,i \end{bmatrix}$

8) $\begin{bmatrix} -7/3 + 1/3\,i & -1 - 5/3\,i \\ -5/6 + 3/2\,i & -7/6 - \frac{11}{6}\,i \end{bmatrix}$

9) $\begin{bmatrix} 4/3 + 2/3\,i & 2/3 \\ -5/2 - 1/6\,i & 1/6 + \frac{17}{6}\,i \end{bmatrix}$

10) $\begin{bmatrix} 2/3\,i & 4/3 - 2/3\,i \\ 1/6 - \frac{17}{6}\,i & 1/6 + 5/2\,i \end{bmatrix}$

11) $\begin{bmatrix} \frac{17}{6} - 1/6\,i & -1/6 + 5/2\,i \\ 2/3\,i & -2/3 + 4/3\,i \end{bmatrix}$

12) $\begin{bmatrix} \frac{17}{6} + 1/6\,i & 5/2 - 1/6\,i \\ 2/3 & 2/3 + 4/3\,i \end{bmatrix}$

13) $\begin{bmatrix} 7/6 + 5/6\,i & \frac{11}{6} - 3/2\,i \\ 1 - 7/3\,i & -5/3 - 1/3\,i \end{bmatrix}$

14) $\begin{bmatrix} -3/2 + 5/6\,i & 7/6 - \frac{11}{6}\,i \\ 7/3 + 1/3\,i & -5/3 - i \end{bmatrix}$

15) $\begin{bmatrix} -1/3 - 7/3\,i & 5/3 - i \\ 3/2 + 5/6\,i & -\frac{11}{6} + 7/6\,i \end{bmatrix}$

16) $\begin{bmatrix} -1/3 - 5/3\,i & -1 - 7/3\,i \\ \frac{11}{6} + 3/2\,i & 5/6 + 7/6\,i \end{bmatrix}$

17) $\begin{bmatrix} -1/6 - \frac{37}{6}\,i & -1/6 + \frac{35}{6}\,i \\ 5/6 + 3/2\,i & -\frac{13}{6} - 3/2\,i \end{bmatrix}$

18) $\begin{bmatrix} 2/3 + 16/3\,i & \frac{20}{3} + 2/3\,i \\ -3/2 - \frac{25}{6}\,i & 7/6 + 3/2\,i \end{bmatrix}$

19) $\begin{bmatrix} -3/2 + \frac{25}{6}\,i & 3/2 + 7/6\,i \\ -16/3 - 2/3\,i & \frac{20}{3} - 2/3\,i \end{bmatrix}$

20) $\begin{bmatrix} -3/2 + 7/6\,i & \frac{25}{6} - 3/2\,i \\ 2/3 - \frac{20}{3}\,i & 16/3 - 2/3\,i \end{bmatrix}$

21) $\begin{bmatrix} -7/3 - i & 5 - 11/3\,i \\ \frac{17}{6} - \frac{35}{6}\,i & \frac{19}{6} + 1/6\,i \end{bmatrix}$

22) $\begin{bmatrix} \frac{17}{6} + \frac{19}{6}\,i & -1/6 - \frac{35}{6}\,i \\ 5 - 7/3\,i & -1 + 11/3\,i \end{bmatrix}$

23) $\begin{bmatrix} -11/3 + i & 5 + 7/3\,i \\ 1/6 - \frac{35}{6}\,i & -\frac{19}{6} - \frac{17}{6}\,i \end{bmatrix}$

24) $\begin{bmatrix} -11/3 - 5\,i & 7/3 + i \\ \frac{19}{6} + 1/6\,i & -\frac{35}{6} - \frac{17}{6}\,i \end{bmatrix}$

25) $BB49 \begin{bmatrix} -1/6 - 5/2\,i & 1/6 - \frac{17}{6}\,i \\ -4/3 + 2/3\,i & 2/3\,i \end{bmatrix}$

26) $\begin{bmatrix} 5/2 + 1/6\,i & -1/6 - \frac{17}{6}\,i \\ -4/3 - 2/3\,i & -2/3 \end{bmatrix}$

27) $\begin{bmatrix} -2/3 - 4/3\,i & 2/3 \\ -5/2 + 1/6\,i & \frac{17}{6} + 1/6\,i \end{bmatrix}$

28) $\begin{bmatrix} -2/3\,i & 2/3 - 4/3\,i \\ -\frac{17}{6} + 1/6\,i & 1/6 - 5/2\,i \end{bmatrix}$

29) $\begin{bmatrix} 5/3 + i & 7/3 + 1/3\,i \\ -7/6 + \frac{11}{6}\,i & -3/2 + 5/6\,i \end{bmatrix}$

30) $\begin{bmatrix} -1 + 7/3\,i & 5/3 + 1/3\,i \\ -7/6 - 5/6\,i & -\frac{11}{6} + 3/2\,i \end{bmatrix}$

31) $\begin{bmatrix} -5/6 - 7/6\,i & \frac{11}{6} + 3/2\,i \\ 1 + 7/3\,i & -1/3 - 5/3\,i \end{bmatrix}$

32) $\begin{bmatrix} -3/2 - 5/6\,i & \frac{11}{6} - 7/6\,i \\ 1/3 + 7/3\,i & -5/3 + i \end{bmatrix}$

33) $\begin{bmatrix} 1/6 - 5/2\,i & -\frac{17}{6} + 1/6\,i \\ 2/3 - 4/3\,i & -2/3\,i \end{bmatrix}$

34) $\begin{bmatrix} \frac{17}{6} + 1/6\,i & -5/2 + 1/6\,i \\ 2/3 & -2/3 - 4/3\,i \end{bmatrix}$

35) $\begin{bmatrix} -2/3 & -4/3 - 2/3\,i \\ -1/6 - \frac{17}{6}\,i & 5/2 + 1/6\,i \end{bmatrix}$

36) $\begin{bmatrix} 2/3\,i & -4/3 + 2/3\,i \\ 1/6 - \frac{17}{6}\,i & -1/6 - 5/2\,i \end{bmatrix}$

37) $\begin{bmatrix} 7/3 + i & 1/3 - 5/3\,i \\ 5/6 - 7/6\,i & -3/2 - \frac{11}{6}\,i \end{bmatrix}$

38) $\begin{bmatrix} -1/3 + 7/3\,i & 1 - 5/3\,i \\ 5/6 + 3/2\,i & -\frac{11}{6} - 7/6\,i \end{bmatrix}$

39) $\begin{bmatrix} -5/6 + 3/2\,i & -7/6 - \frac{11}{6}\,i \\ -7/3 + 1/3\,i & -1 - 5/3\,i \end{bmatrix}$

40) $\begin{bmatrix} -3/2 + \frac{11}{6}\,i & -7/6 + 5/6\,i \\ -5/3 + 1/3\,i & -7/3 + i \end{bmatrix}$

41) $\begin{bmatrix} -3/2 - 7/6\,i & 3/2 - \frac{25}{6}\,i \\ -\frac{20}{3} + 2/3\,i & 16/3 + 2/3\,i \end{bmatrix}$

42) $\begin{bmatrix} \frac{25}{6} - 3/2\,i & -3/2 + 7/6\,i \\ 16/3 - 2/3\,i & 2/3 - \frac{20}{3}\,i \end{bmatrix}$

43) $\begin{bmatrix} 2/3 - 16/3\,i & 2/3 + \frac{20}{3}\,i \\ \frac{25}{6} + 3/2\,i & 7/6 - 3/2\,i \end{bmatrix}$

44) $\begin{bmatrix} \frac{20}{3} + 2/3\,i & 2/3 + 16/3\,i \\ 7/6 + 3/2\,i & -3/2 - \frac{25}{6}\,i \end{bmatrix}$

45) $\begin{bmatrix} 5 + 11/3\,i & 7/3 - i \\ \frac{19}{6} - 1/6\,i & -\frac{17}{6} - \frac{35}{6}\,i \end{bmatrix}$

46) $\begin{bmatrix} -1/6 + \frac{19}{6}\,i & -\frac{35}{6} - \frac{17}{6}\,i \\ -11/3 - 5\,i & 1 - 7/3\,i \end{bmatrix}$

47) $\begin{bmatrix} \frac{35}{6} + 1/6\,i & \frac{17}{6} - \frac{19}{6}\,i \\ 1 + 11/3\,i & 7/3 - 5\,i \end{bmatrix}$

48) $\begin{bmatrix} -1/6 + \frac{19}{6}\,i & \frac{17}{6} - \frac{35}{6}\,i \\ -5 + 11/3\,i & 1 - 7/3\,i \end{bmatrix}$