Calculus of Variations and Optimal Control

Optimization problems occur in science, technology and everyday life. For instance, which plane figure of a given perimeter encloses the largest area? Which trajectory gets a spaceship into orbit around Earth using the minimum amount of fuel? How to devise the cheapest well-balanced diet using given foods? From a mathematical point of view, optimization problems reduce to finding a maximum or a minimum of a function on a certain (probably infinite dimensional) space. In this course, we study mathematical methods for solving optimization problems of different nature.

Program of the course

0. Reminder: the method of Lagrange multipliers, normed spaces.

1. Calculus of variations: history of optimization problems, Euler and Euler–Lagrange equations, classical problems in the calculus of variations (such as isoperimetric, brachistochrone and Newton problems).

2. Optimal control: Pontryagin maximum principle and Hamilton–Jacobi–Bellman equation.

3. Convex analysis and geometry: mixed volumes and main inequalities of convex geometry (such as isoperimetric, Brunn–Minkowski and Alexandrov–Fenchel inequalities), simplex method, linear and convex programming.

4. Introduction to optimal transport and geometric applications (geodesics and minimal surfaces).

5. Introduction to game theory.