Experimental Mathematics

Course description: In this course, we learn how to use *Mathematica* in various mathematical problems. *Mathematica* is great for visualizing mathematical objects (such as functions, sets, polytopes etc.), for collecting empirical data and for testing conjectures. In particular, we focus on 3D graphics tools provided by *Mathematica*. These tools are ideal for drawing high precision pictures for mathematical papers.

What is *Mathematica*? *Mathematica* combines a computer algebra system and a high-level programming language. It is a commercial product developed by Wolfram Research. There are free alternatives to *Mathematica* such as Sage and other commercial alternatives such as Maple. There are two reasons for choosing *Mathematica* for our course: first, *Mathematica* is already installed on many department computers, second, *Mathematica* has special features (such as manipulator) that can be used to display results nicely. If you master Mathematica you will have no difficulty working with other computer algebra systems since they are all build on the same principles.

Topics of the course

- 1. How to use *Mathematica*: syntax and grammar; lists, matrices and tables; manipulator.
- 2. Graphics in *Mathematica*: function plots; parameteric plots; polygons and polytopes.
- 3. Numbers: decimal and continued fractions; arithmetic operations.
- 4. Functions: elementary and special functions; differentiation and integration; Taylor series.
- 5. Algebra: polynomials and algebraic equations; discriminants and resultants; rational functions and simple fractions; generating functions; linear operators and normal forms.
- 6. Differential equations: linear ordinary differential equations, Riccati equations, first order partial differential equations.