**Convex Geometry**

*Learning seminar organized by Alexander Esterov, Valentina Kiritchenko, Alexander Kolesnikov, Evgeny Smirnov*

Convex geometry and geometry of polytopes are closely related with almost all branches of contemporary mathematics including algebra, geometry, combinatorics, topology, analysis and mathematical physics. The goal of the seminar is to learn various classical results and notions of convex geometry (such as Brunn–Minkowski inequality, Dehn–Sommerville relations, mixed volumes, Ehrhart polynomial) and to study applications of these results in toric and tropical geometry, analysis, number theory, representation theory and algebraic geometry.

**Topics of the seminar**

1. Convex geometry and analysis: mixed volumes; isoperimetric, Brunn–Minkowski and Alexandrov–Fenchel inequalities; Alexandrov–Bakelmann–Pucci maximum principle.

2. Convex polytopes and toric geometry: Kushnirenko and Bernstein theorems on the number of solutions of polynomial equations with given Newton polytopes; virtual polytopes and finitely additive measures on polytopes; Ehrhart polynomials and multidimensional Pick’s formula; toric varieties.

3. Tropical and real algebraic geometry: patchworking; amoebas; tropical curves, polynomials and Bezout theorem; Mikhalkin’s theorem on counting algebraic curves; secondary and fiber polytopes.

4. Theory of Newton–Okounkov bodies: approximation theorem for semigroups of integer points; graded algebras of rational functions and geometric valuations; convex-geometric proof of Hodge inequality for algebraic surfaces, algebro-geometric proof of Alexandrov–Fenchel inequalities.

5. Geometry and combinatorics of convex polytopes: Dehn–Sommerville relations; classification of regular polytopes in Euclidean spaces.

6. Convex polytopes and representation theory: permutohedra and associahedra; Gelfand–Zetlin bases and polytopes.

7. Convex geometry and number theory: multidimensional continued fractions, sails and Klein polyhedra; Oppenheim conjecture.