Topology 2: brief description and syllabus

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The main purpose of the course is to give a careful introduction to singular homology and cohomology. The main prerequisites are some basic algebra (groups, rings, fields), topology (topological and metric spaces, continuous maps, homotopy between continuous maps, coverings and the fundamental group) and category theory (nothing fancy, just the notions of categories, functors and natural transformations). These topics were covered during Semester 1 at MiM but we will recall some or all of them if necessary. A tentative syllabus is as follows.

- Introduction. What is algebraic topology and what is it good for?
- Singular homology. Basic homological algebra: exact sequences, complexes, 5-lemma, homotopy.
- Homological algebra continued: acyclic models. The acyclic model theorem is an abstract but very powerful tool that comes in handy whenever one has to prove that two things have isomorphic homology. We will introduce it early on and later it will save us a lot of time.
- Mayer-Vietoris and excision for singular cohomology. These are the tools that will allow us to compute the singular homology of most spaces that we encounter in everyday life.
- CW-complexes; the Schubert decompositions of the Grassmannians. A CW-structure is a way to decompose a space into simple pieces called cells. Most of the spaces algebraic topology deals with have a CW-structure or at least are homotopy equivalent to CW-complexes.
- Cellular cohomology. The singular homology of a CW-complex can be computed using a very "small" complex. But like everything else, this comes at a price.
- Homology and cohomology with coefficients. The universal coefficient theorems. These theorems allow one to express, non-canonically, the (co)homology with coefficients in terms of the integral homology.
- Cup and cap products. As opposed to homology, the cohomology with ring coefficients is again a ring. This fact has many geometric consequences.
- The Künneth isomorphisms. These allow one to compute the (co)homology of the product of two spaces if one knows the (co)homology of the factors.
- Topological manifolds and the Poincaré duality. Manifolds are spaces that locally look like the Euclidean space. The cohomology of a manidold has a hidden symmetry called the Poincaré duality.
- Lefschetz theorems. The contribution of a nondegenerate fixed point in the manifold case. In many cases one can conclude that a self-map of a compact space has a fixed point by looking at the induced map in cohomology. In the manifold case one has a more precise result.

Alternatively, if it turns out that everyone is familiar with these topics already, we could cover some of the more advanced ones, such as spectral sequences, characteristic classes or the topological K-theory.

Tle lectures in the second semester 2012-2013 take place at HSE on Monday at 3.30 pm, room 311. The final mark is calculated as follows. HSE students take two exams, one at the end of each of the modules 3 and 4, which contribute 55% towards the final mark for the respective module, while problems contribute 45%. For Math in Moscow students the rules will be different. The final exam and class test (which will take place around week 7) are worth respectively 60% and 20% of the final mark. Another 40% can be earned by solving problems and explaining the solutions to tutors during problem classes (Mon 5 pm, same room as lectures). The lecturer's office hours are 6.30 - 7.50 pm on Monday and 3.30-4.50 pm on Friday (HSE, room 1006), or by appointment.

The main references are

- *A Course in Homotopy Theory* by D. Fuchs and A. Fomenko.
- *Algebraic Topology* by A. Hatcher, freely available online at http://www.math.cornell.edu/~hatcher/AT/ATpage.html.
- *Characteristic classes* by J. Milnor and J. Stasheff.

Occasionally we'll be using other sources as well.