Functional Analysis 2

(Spring 2014)

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This is a continuation of Functional Analysis 1.

Syllabus[1]

1. **Compact and Fredholm operators.** Compact and Fredholm operators, basic properties and examples. The Fredholm index. The Riesz-Schauder theory. The Fredholm Alternative. Properties of the spectrum of a compact operator. Applications to integral equations. The Nikolski-Atkinson criterion. The Calkin algebra. The continuity of the index. The stability of the index under compact perturbations. The essential spectrum. Applications to Toeplitz operators.

2. **Compact operators on a Hilbert space.** Hilbert space operators and their adjoints. Spectra of unitary and selfadjoint operators. The Hilbert-Schmidt theorem on the diagonalization of compact selfadjoint operators. The Schmidt theorem on the structure of compact operators. Applications to the Sturm-Liouville problem. Hilbert-Schmidt operators and nuclear operators. The trace.

3. **Topological vector spaces.** Locally convex spaces. Examples. Continuous linear operators. Normability and metrizability criteria. Dual pairs and weak topologies. The bipolar theorem and corollaries. The Banach-Alaoglu theorem. Weak topologies and compact operators.

4. **Distributions.** Operations on distributions. The sheaf of distributions. The support. Compactly supported distributions and tempered distributions. Tensor product and convolution of distributions. Structure theorems for distributions.

5. **The Fourier transform.** The Fourier transform of integrable functions on the integers, on the circle, and on the real line. Basic properties of the Fourier transform on the real line. The Fourier transform as an automorphism of the Schwartz space. The Fourier transform of tempered distributions. The L^2-Fourier transform. The Plancherel Theorem.

6. **The Spectral Theorem.** The continuous and Borel functional calculi for a selfadjoint operator. Spectral measures and representations of algebras of continuous functions. The Spectral Theorem and the functional model for a selfadjoint operator.

[1] Items 5 and 6 may be partially omitted.