Elliptic Functions

Special course, spring semester 2013-2014
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This course is for third/fourth year students, master course students and aspirant course students. In the presence of those who do not understand Russian, the lecture will be in English.

Basic prerequisites: calculus and complex analysis.

An elliptic function is defined as a doubly periodic meromorphic function on the complex plane. The theory of elliptic functions was born in the eighteenth century. At first Fagnano, Euler, Legendre, Gauss and others studied elliptic integrals. Then in the nineteenth century Abel and Jacobi changed the viewpoint and studied the inverse functions of elliptic integrals, namely, elliptic functions, which was a great advance. Riemann and Weierstrass developed the theory further. The theory of elliptic functions thus founded is a prototype of today's algebraic geometry.

On the other hand, elliptic functions appear in various problems in mathematics as well as in physics. In fact the name "elliptic integral" comes from calculation of length of an arc of an ellipse.
Other examples are:

- The arithmetic-geometric mean is closely related to the elliptic integral.
- Solutions of various problems in physics – motion of a pendulum, form of skipping rope, motion of a top (= a rigid body), KdV equation (typical integrable non-linear differential equation), … – are expressed in terms of elliptic functions and elliptic integrals.
- One can solve a general fifth order equation, using an elliptic function.

In this course we shall put emphasis on analytic aspects and applications of elliptic functions. The following subjects are planned to be discussed:

Length of arcs of curves (ellipse, sine curve, lemniscates, …);
Elliptic integrals and their normal forms;
Elliptic functions;
Theta functions and their properties;
Properties of elliptic functions;
Applications of elliptic functions and elliptic integrals.