Several Complex Variables. Program of theoretical work on March 23, 10h30–12h00

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1. Holomorphic functions in several complex variables: definition, Cauchy–Riemann equations. Equivalent definitions in terms of Cauchy–Riemann equations and separate holomorphicity. Hartogs' separate holomorphicity theorem (without proof).

2. Multidimensional Cauchy formula. Osgood Lemma. Taylor series.

3. Convergence of power series. Abel's lemma. Convergence polydisk, convergence multiratdius. Convergence domain: invariance under the action of torus $\mathbb{T}^n = S^1 \times \ldots S^1$ by multiplication of the coordinates by complex number with unit module.

4. Multidimensional version of Cauchy–Hadamard formula for convergence multiraduis. Differentiation of converging series: formula for Taylor series coefficients in terms of partial derivatives and factorials.

5. Theorem on uniqueness of analytic extension. Openness Principle, Maximum Principle, Liouville Theorem.

6. Hartogs' Theorem on analytic extension from a "Hartogs' Figure" to polydisk. Theorem on erasing compact singularities: proof for polydisk.

7. Analytic sets: definition and main properties (regular and singular parts, irreducible components, stratification) without proofs. Holomorphic extension of bounded holomorphic functions in the complement of an analytic subset.

8. Weierstrass polynomials. Definition and Weierstrass Preparatory Theorem.

9. Factorization of germs of Weierstrass polynomials and analytic functions as product of irreducible ones.

10. Characterization of irreducible Weierstrass polynomials in terms of the geometry of their zero loci as coverings.

11. Holomorphic Implicit Function Theorem. Irreducible germs of holomorphic functions: characterization of singular points of its zero locus as

zeros of differential.

12. Local holomorphic parametrization of zero locus of irreducible germ of holomorphic function in two variables.

13. A necessary condition for irreducibility of germ of holomorphic function in two variables in terms of lower homogeneous part.

14. Irreducibility of an analytic set with a connected regular part (with proof). Theorem on irreducibility and connectivity of regular part (statement without proof).

15. Irreducibility of zero locus of an irreducible germ of holomorphic function.

16. Uniqueness of decomposition of an analytic set as a union of irreducible ones. Dimension of analytic set. Theorem on presentation of a germ of analytic set of pure dimension k as a covering over a k-dimensional polydisk (without proof).

17. Remmert Proper Mapping Theorem (statement without proof). Germs of hypersurfaces: principality of ideal and generating function.

18. Intersection index of a hypersurface and a one-dimensional complex submanifold (local and global version). Invariance under C^1-deformations.

19. Chow Theorem.

20. Biholomorphic automorphisms of projective space.

21. Holomorphic mappings of domains in normed complex spaces. Generalized Maximum Principle.

22. Generalized Schwarz Lemma for holomorphic mappings of balls in normed complex spaces.

23. Cauchy Inequality for Taylor coefficients of bounded holomorphic functions.

24. Cartan's Theorem on automorphisms of bounded domains.

25. Non-isomorphicity of ball and polydisk.

26. Automorphisms of the ball.

27. Automorphisms of the polydisk.