Several Complex Variables. Final exam.

Wednesday, April 27, 2016.

1. Domains of holomorphy. Holomorphically convex hulls. Holomorphically convex domains. Main definitions. Oka Theorem, part 1: each holomorphically convex domain is a domain of holomorphy.

2. Cartan–Thullen Theorem on gap between holomorphically convex hull of a compact set and the ambient domain.

3. Oka's Theorem, part 2: each domain of holomorphy is holomorphically convex.

4. Continuity Principle. Levi convexity.

5. Levi form. Necessary and sufficient Levi convexity conditions for domains with C^2-smooth boundary.

6. (Pluri)harmonic (pluri)subharmonic functions: main definitions and properties. L-convexity of the domain $\phi < 0$ in $\mathbb{C}^n$, where ϕ is a plurisubharmonic function.

7. Stein manifolds. Definitions and examples. Embedding Theorem (without proof).

8. Riemann domains and their holomorphic extensions: main definitions. Existence and uniqueness of holomorphic extension.

9. One-dimensional $\bar{\partial}$-problem: solution for unit disk.

10. Dolbeault cohomology. Their triviality for polydisk ($\bar{\partial}$-Poincaré Lemma).

11. Holomorphic hypersurfaces. Problem of the existence of global defining holomorphic function. Reduction to Cousin Problem.

12. Solution of Additive Holomorphic Cousin Problem for polydisk.

13. Sheaves and their cohomology: basic definitions and properties.

14. Short exact sequence of complexes and long cohomological exact sequence.

15. Triviality of the cohomology $H^1(\Delta, \mathcal{O}^*)$ for polydisk Δ. Corollary: existence of global defining holomorphic function for every hypersurface.

16. Extension theorem: each holomorphic function on a hypersurface in a polydisk is the restriction to it of a global holomorphic function.

17. Stein manifolds. Cartan's A and B Theorems (without proofs).