Workshop "Integrable Systems, Random Matrices, Algebraic Geometry and Geometric Invariants"

Математический институт РАН им В.А.Стеклова, комн. 530
V. A. Steklov Mathematical Institute, room 530.

20 – 22 февраля/February 2012

Organisers: Leonid Chekhov (Steklov Mathematical Institute and Laboratoire Poncelet), Takashi Takebe (National Research University – Higher School of Economics)

Программа семинара / Seminar programme:

Monday, 20 February:

11:00 – 11:40 L. Chekhov: Quantum Riemann surfaces related to (quantum) Liouville theory.
12:00 – 12:40 A. Zotov: Stationary and Non-Stationary Baxter Equations for Gaudin-Painlevé Models.
14:00 – 14:40 A. Zabrodin: Classical tau-function for quantum spin chains.
15:00 – 15:40 K. Hasegawa: Quantizing the discrete Painlevé VI equation.

Tuesday, 21 February:

10:00 – 10:40 E. Morrison: Modular Frobenius manifolds and Their Flows.
11:00 – 11:40 A. Ikeda:The space of stability conditions for quivers with two vertices and almost Frobenius structures.
12:00 – 12:40 I. Strachan: Elliptic V-systems and solutions to the WDVV equations.
14:00 – 14:00 J. Shiraishi: Quantum Algebraic Approach to Refined Topological Vertex.
15:00 – 15:40 T. Kuwabara: Localization of affine W-algebras at critical level.

Wednesday, 22 February:

10:00 – 10:40 S. Iwao: Singular curve and discrete integrable system.
11:00 – 11:40 S. Yanagida: On Hall algebra of complexes.
12:00 – 12:40 S. Kondo: Mirahoric representations and the local L-factor.
14:00 – 14:40 A. Mironov: Character expansions and integrability: from Hurwitz to HOMFLY.
15:00 – 15:40 S. Lando: On stratification of the range of a generic function having only A-singularities.

Thursday, 23 February:

Free discussions

Abstracts

Leonid Chekhov (Steklov Mathematical Institute and Laboratoire Poncelet):

Quantum Riemann surfaces related to (quantum) Liouville theory.

We generalize the technique developed for polynomial potentials and inspired by beta-model integrals to the case of logarithmic potentials related to n-point conformal blocks for correlation functions of quantum Liouville theory.

Andrei Zotov (Institute for Theoretical and Experimental Physics):

Stationary and Non-Stationary Baxter Equations for Gaudin-Painlevé Models.

We present two algebraic constructions for quantization of spectral curves of (the reduced) Gaudin models and related Painlevé equations. The first one is the Quantum Painlevé-Calogero Correspondence. It represents the classical linear problem for Painlevé equation (non-autonomous version of the Gaudin model) in a form of its quantum version — non-stationary Schrödinger equation. The second construction is based on bispectral-like duality between $gl(N)$ Gaudin model and N-site Heisenberg chain. We describe this duality on classical and quantum levels.

Anton Zabrodin (ITEP, Institute of Biochemical Physics and NRU-HSE):

Classical tau-function for quantum spin chains.

For any generalized quantum integrable spin chain (or integrable lattice model of statistical mechanics) we introduce a master T-operator which is a sort of generating function for commuting quantum transfer matrices constructed by means of the fusion procedure in the auxiliary space. We show that the functional relations for the transfer matrices are equivalent to an infinite set of model-independent bilinear equations of the Hirota form for the master T-operator, which allows one to identify it with τ-function of an integrable hierarchy of classical soliton equations. We will consider spin chains with rational $GL(N)$-invariant R-matrices but the result is independent of a particular functional form of the transfer matrices and directly applies to quantum integrable models with more general (trigonometric and elliptic) R-matrices and to supersymmetric spin chains.

Koji Hasegawa (Tohoku University):

Quantizing the discrete Painlevé VI equation

We quantize the discrete Painlevé VI equation which is obtained by Jimbo and Sakai. There are two ways to quantize: One is based on the affine Weyl group symmetry of the equation, the other is Lax formalism or the monodromy preserving deformation point of view. It turn out that both approaches are succesful and gives the same result.

Ewan Morrison (University of Glasgow):

Modular Frobenius manifolds and Their Flows.

The structure of a Frobenius manifold was introduced by Dubrovin as a way to give geometric understanding to the Witten-Dijkgraaf-Verlinde-Verlinde (WDVV) equations of topological field theory. Associated to a given Frobenius manifold is a hierarchy of partial differential equations of hydrodynamic type called the principle hierarchy. Firstly I will review the correspondence between solutions of the WDVV equations and Frobenius manifolds, and the construction of the principle hierarchies. Secondly, I will look at an involutive symmetry defined on the space of Frobenius manifolds and present how this lifts to the principle hierarchies. For a certain subclass of Frobenius manifold lying at fixed points of the symmetry - which we call modular Frobenius manifolds - this leads to equations of hydrodynamic type that inherit the modular symmetry from the Frobenius manifold. This talk is based on joint work with my supervisor, Prof. Ian Strachan.

Akishi Ikeda (the University of Tokyo):

The space of stability conditions for quivers with two vertices and almost Frobenius structures.

The space of stability conditions on triangulated categories was introduced by T. Bridgeland. It is important to find some geometric structures on these spaces. In this talk, we construct the almost Frobenius structure (without assuming the existence of one of axioms of almost Frobenius structure) on some open subset of the space of stability conditions on the derived category of the finite dimensional representations of the quiver with two vertices and n arrows.

Ian Strachan (University of Glasgow):

Elliptic V-systems and solutions to the WDVV equations.

There are two recurring ideas in the theory of integrable systems - the appearence of root systems, and a sequence of generalizations, starting with rational models and moving to trigonometric to elliptic generalizations. In this talk a class of elliptic solutions to the WDVV equations will be described in terms of the elliptic trilogarithm functions and a set of vectors which define a so-called elliptic V-system. This provides a generalization of a V-system first introduced by Veselov in connection to rational solutions of the WDVV equations.

Jun-ichi Shiraishi (the University of Tokyo):

Quantum Algebraic Approach to Refined Topological Vertex.

We establish the equivalence between the refined topological vertex of Iqbal-Kozcaz-Vafa (or Awata-Kanno) and a certain representation theory of the quantum algebra introduced by Miki. Our construction involves trivalent intertwining operators associated with triples of the bosonic Fock modules. Resembling the topological vertex, a triple of vectors is attached to each intertwining operator, which satisfy the Calabi-Yau and smoothness conditions. (Joint work with H. Awata and B. Feigin. arXiv:1112.6074)

Toshiro Kuwabara (National Research University – Higher School of Economics):

Localization of affine W-algebras at critical level.

The affine W-algebras are vertex algebras which generalize the Virasoro algebra and are constructed by Drinfeld-Sokolov reduction of affine vertex algebras. We construct a sheaf of vertex algebras on the arc space (the infinite-Jet sheme) of the Slodowy variety, called ACDO (Asymptotic Chiral Differential Operators), and show that the vertex algebra of its (C^*-invariant) global sections is isomorphic to the corresponding affine W-algebra at critical level. This talk is based on a joint work with T. Arakawa and F. Malikov. (arXiv:math/1112.0089)

Shinsuke Iwao (Rikkyo University):

Singular curve and discrete integrable system.

It is known that the general solution of the discrete KdV equation with periodic boundary condition is expressed as a ratio of the theta functions associated with an algebraic curve, which is called the spectral curve. In this talk, the general solutions of the discrete KdV equation with more complicated boundary conditions are introduced. The solutions are associated with singular spectral curves.

Shintaro Yanagida (Kobe University):

On Hall algebra of complexes.

The topic of my talk is the Hall algebra of complexes, which is recently introduced by T. Bridgeland. I will discuss its properties and relation to auto-equivalences of derived category.

Satoshi Kondo (IPMU, the Univeristy of Tokyo):

Mirahoric representations and the local L-factor

Let k be a nonarchimedean local field, O be the ring of integers, and $\wp$ be the maximal ideal. Let n be a nonnegative integer. We set K_n to be the subgroup of elements $g = (g_{ij})$ of $GL(d, O)$ such that the last row $(g_{d1}, \ldots, g_{dd})$ is congruent to $(0, \ldots, 0, 1)$ modulo $\wp^n$. We call the subgroups K_n mirahoric. Let us call an irreducible admissible representation (π, V) of $GL(d, k)$ mirahoric if there exists a nonnegative integer n such that the K_n-fixed part V^{K_n} is nonzero. For example, all generic representations are mirahoric since there always exists a "new vector" in a generic representation (Jacquet-Piatetskii-Shapiro-Shalika).
In this talk, we give various characterizations of mirahoric representation, using the local Langlands correspondence, the Zelevinsky involution, and the classification using multisegments. We also show that, given an irreducible admissible representation π of conductor c, there exists a unique mirahoric representation of conductor c in the Jordan-Hölder constituents of the induced representation (the standard module) attached to π in the classification theorem. Moreover, the multiplicity is one. This property is analogous to that of a generic representation.

Andrei Mironov (Lebedev Physics Institute and ITEP):

Character expansions and integrability: from Hurwitz to HOMFLY.

Character expansions emerge in various applications in mathematical physics, from generating functions of Hurwitz numbers to knot theory. They sometimes happen to describe a τ-function of KP/Toda hierarchy, however, generically they have to correspond some extensions of the standard τ-function. This pose a problem of constructing such extensions. The talk explains the problem, but does not answer the question.

Sergei Lando (National Research University – Higher School of Economics):

On stratification of the range of a generic function having only A-singularities.

Let $F : M \to N$ be a generic finite holomorphic map of two compact complex manifolds of the same dimension. Suppose also that F has singularities of the types A_k, $k = 1, 2, 3, \dots$ only. Then the target manifold N is split into strata of the form $A_{n_1,\dots,n_d}(N)$ consisting of points at whose preimages the function F has singularities of types $A_{n_1}, \dots, A_{n_d}$. According to Kazarian's principle, the generating function for the cohomology classes of the closures of the submanifolds $A_{n_1,\dots,n_d}(N)$ can be expressed as the exponent of F_* applied to certain universal generating function whose coefficients are relative Chern classes of F. We claim that the latter function is a solution to the Kadomtsev–Petviashvili integrable hierarchy of partial differential equations.