

QUANTUM ALGEBRAS

This course is a HIC for master course and 3rd and 4th years of undergraduate. The prerequisite knowledge is linear algebra and basic algebra. Note that this course is given in English.

In this course, we study (mainly combinatorial) representation theory of quantum enveloping algebras (quantum groups) of type $A_{r-1}^{(1)}$ and cyclotomic Hecke algebras (Ariki-Koike algebras).

The cyclotomic Hecke algebras are Iwahori-Hecke algebras (q -deformation) associated with the wreath products $(\mathbb{Z}/m\mathbb{Z}) \wr S_n = (\mathbb{Z}/m\mathbb{Z})^n \rtimes S_n$ and generalize the Iwahori-Hecke algebras of type A and B . The quantum enveloping algebras are q -deformation of universal enveloping algebras of the Lie algebras. These algebras are widely known by their rich combinatorial structures such as Young diagrams and Kashiwara's crystals and by many applications to other areas (integrable systems, knot invariants, etc.)

The primary goal of the course is to introduce the representation theory of quantum enveloping algebras and cyclotomic Hecke algebras. Combinatorial objects such as Young diagrams, Young tableaux and crystal structures are the main tool of our study. The secondary goal is to study a beautiful connection between modular representation theory of cyclotomic Hecke algebras and the crystal associated with an irreducible module of a quantum enveloping algebra, which we call the LLTA theory. This theory was established in 1990s and quite recently it motivated development of categorification theory.

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