GROEBNER BASIS THEORY

This course is a спецкрус for 1st and 2nd years of undergraduate. Note that the lecture is given in English. Participants are required no prerequisite knowledge but highschool algebra such as basic computation of polynomials.

The main topic of this спецкрус is the structure of ideals of polynomial rings in many variables over the complex number field. We will start from reviewing the structure of the polynomial ring in one variable, $\mathbb{C}[x]$. The degree of polynomials gives an Euclidean ring structure, and arbitrary ideal of $\mathbb{C}[x]$ is generated by a single polynomial. By this fact, the structure of ideals of the polynomial ring in one variable is easy to control.

On the other hand, the polynomial ring in many variables is not so easy to investigate. In general, ideals are generated by many polynomials, not a single, and there are many choices of generators. In the last century, B. Buchberger established the theory of Groebner (Gröbner) basis which gave "standard" generators for a ideal of the polynomial ring. Moreover, he gave an algorithm how to compute the Groebner basis for a given ideal. Now this algorithm is widely used by software of algebraic computing systems.

In this course, we study the theoretical aspect of the Groebner basis. We study the algorithm to compute the Groebner basis and how it is useful to investigate the structure of ideals. We will not care how to use software of algebraic computing systems. Thus the skills of using computers is not required.

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