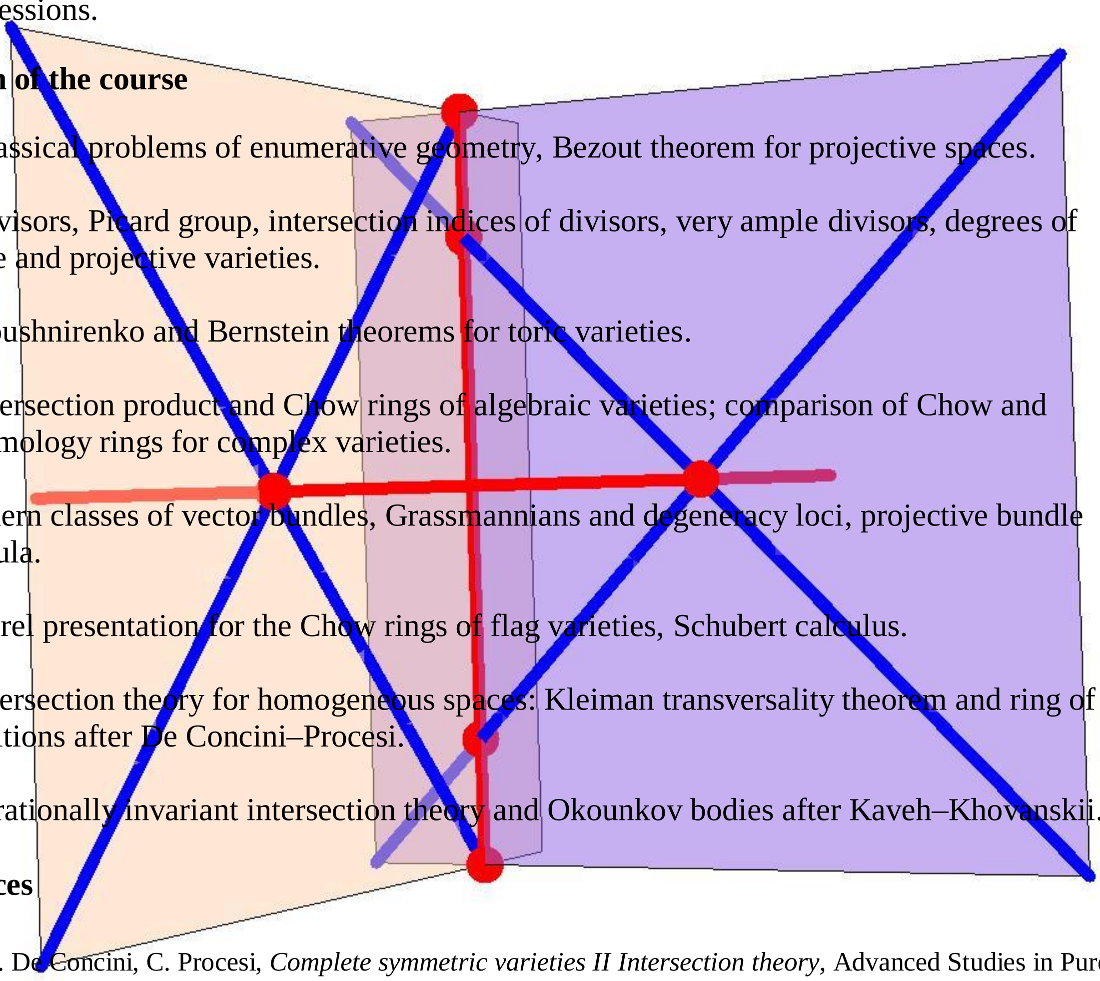
# Intersection Theory

How many lines in 3-space intersect 4 given lines? How many smooth conics are tangent to 5 given conics? These and many other classical problems of enumerative geometry were solved heuristically in the 19th century. Intersection theory was developed in the 20th century in order to provide a rigorous foundation for these solutions. For instance, the first question reduces to computing the intersection indices of hypersurfaces in the Grassmannian $G(2,4)$.

In this course, we study main notions and tools of algebraic intersection theory such as Chow rings and Chern classes. We focus on concrete meaningful examples that can be computed explicitly. In particular, we study intersection theory on toric and flag varieties. The course will be accompanied by the research seminar Convex Geometry and Intersection Theory and problem solving sessions.



**Program of the course**

1. Classical problems of enumerative geometry, Bezout theorem for projective spaces.

2. Divisors, Picard group, intersection indices of divisors, very ample divisors, degrees of affine and projective varieties.

3. Koushnirenko and Bernstein theorems for toric varieties.

4. Intersection product and Chow rings of algebraic varieties; comparison of Chow and cohomology rings for complex varieties.

5. Chern classes of vector bundles, Grassmannians and degeneracy loci, projective bundle formula.

6. Borel presentation for the Chow rings of flag varieties, Schubert calculus.

7. Intersection theory for homogeneous spaces: Kleiman transversality theorem and ring of conditions after De Concini–Procesi.

8. Birationally invariant intersection theory and Okounkov bodies after Kaveh–Khovanskii.

**References**


- C. De Concini, C. Procesi, *Complete symmetric varieties II Intersection theory,* Advanced Studies in Pure Mathematics, **6** (1985), Algebraic groups and related topics, 481-513
- W. Fulton, *Intersection theory*, Second edition. Springer-Verlag, Berlin, 1998
- Ph. Griffiths, J. Harris, *Principles of algebraic geometry.* Reprint of the 1978 original. Wiley Classics Library. John Wiley and Sons, Inc., New York, 1994; Chapter on Chern classes as degeneracy loci
- A. Khovanskii, K. Kaveh, *Newton convex bodies, semigroups of integral points, graded algebras and intersection theory*, Ann. of Math.(2), **176** (2012), 2, 925-978
- S. Kleiman, *Problem 15: Rigorous foundation of Schubert's enumerative calculus, Mathematical Developments arising from Hilbert Problems,* Proc. Symp. Pure Math., **28**, Amer. Math. Soc. (1976), pp. 445-482
- L. Manivel, *Symmetric functions, Schubert polynomials and degeneracy loci.* SMF/AMS Texts and Monographs 6, 2001; Chapter on Grassmannians and flag varieties
- I. R. Shafarevich, *Basic algebraic geometry. 1. Varieties in projective space.* Second edition. Springer-Verlag, Berlin, 1994; Chapter on intersection indices