Geometry of Spherical Varieties

The main goal of this course is to introduce the best-known classes of spherical varieties (such as flag varieties, wonderful compactifications and horospherical varieties) and to study their geometry. The course will be accompanied by the research seminar Convex Geometry and Intersection Theory and problem solving sessions.

Program of the course

1. Motivating examples: Grassmannians, toric varieties, complete conics.

2. Spherical varieties from a geometric viewpoint: log-homogeneous varieties; spherical varieties from representation-theoretic viewpoint: multiplicity-free homogeneous spaces.

3. Flag varieties; Schubert cells, varieties and cycles; cohomology rings of flag varieties: Borel presentation; Schubert calculus: Pieri–Chevalley formula, divided difference operators; Bott–Samelson resolutions of Schubert varieties.

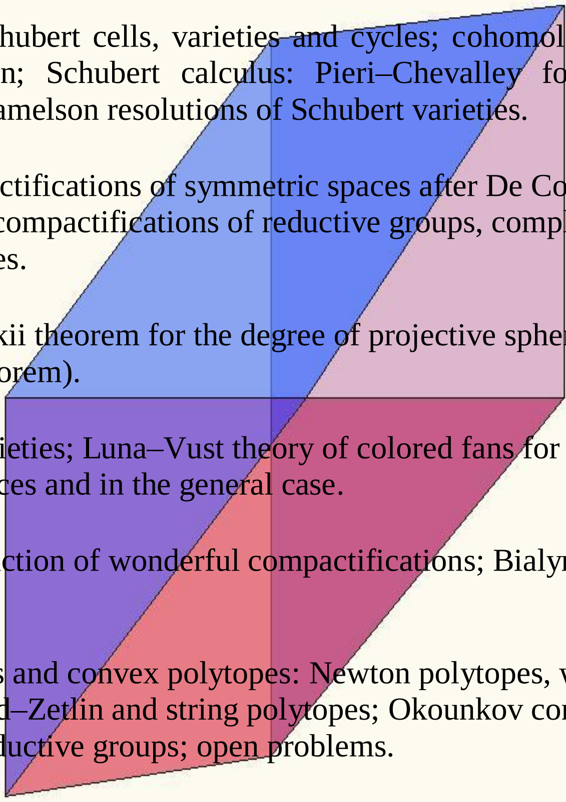
4. Wonderful compactifications of symmetric spaces after De Concini–Procesi, complete quadrics; regular compactifications of reductive groups, complete collineations; symmetric varieties.

5. Brion–Kazarnovskii theorem for the degree of projective spherical varieties (extension of Koushnirenko theorem).

6. Horospherical varieties; Luna–Vust theory of colored fans for horospherical homogeneous spaces and in the general case.

7. Demazure construction of wonderful compactifications; Bialynicki-Birula cell decomposition.

8. Spherical varieties and convex polytopes: Newton polytopes, weight and moment polytopes, Gelfand–Zetlin and string polytopes; Okounkov convex bodies associated with actions of reductive groups; open problems.



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