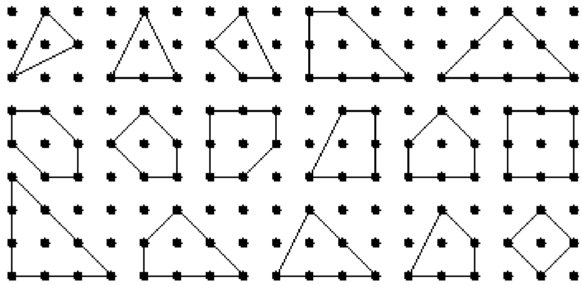
TORIC VARIETIES (2^{nd} module)

Every convex polytope with integer coordinates of the vertices gives rise to an algebraic variety, which is called toric. This is a remarkable correspondence between polytopes and varieties: it always translates important convex-geometric properties and questions to important algebro-geometric ones, and vice versa. For instance, here is the list of all polygons such that the correpsonding toric surfaces are del Pezzo:



One can readily verify that these are exactly the 16 lattice polygons, containing a unique lattice point in their interior. The platonic dual to any of these polygons is again a polygon on this list, and this convex-geometric symmetry corresponds to the algebro-geometric mirror symmetry for del Pezzo surfaces.

The aim of the course is to study this correspondence between polytopes and varieties and its applications to both convex and algebraic geometry. In particular, applying convex geometry to the study of algebraic geometry, we shall obtain far-reaching generalizations of the classical Bezout theorem (the number of common roots of two polynomials of two variables does not exceed the product of their degrees). In the other direction, applying algebraic geometry to the study of convex geometry, we shall describe relations between the numbers of faces of given dimensions in an arbitrary simplicial polytope.

Program: polytopes, fans and toric varieties; convex-algebro-geometric dictionary; Kouchnirenko theorem; mixed volumes; Bernstein theorem; Dehn–Sommerville equations.

References: Textbooks and works by Kouchnirenko, Bernstein, Khovanskii and Timorin at http://math.hse.ru/nis-12-vgeom

Prerequisites: The course can be taken by 2^{nd} year students and higher, and will be accompanied by the research seminar Convex Geometry and Intersection Theory. Attending the course Patchworking in the 1^{st} module will facilitate the understanding (especially for a 2^{nd} year student), but is not necessary.