Quasiconformal mappings, complex dynamics and moduli spaces

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Poincaré–Köbe Uniformization Theorem is a fundamental theorem of geometry saying that each simply-connected Riemann surface is conformally-equivalent to either the Riemann sphere, or complex line $\mathbb{C}$, or unit disk. The quasiconformal mapping theory was founded by H.Grötzsch, M.A.Lavrentiev and C.Morrey Jr. in 1920-1930-ths and later developed by L.Ahlfors and L.Bers. It extends the Uniformization Theorem for *non-constant* and even *discontinuous* complex structures. It has many important applications in various branches of mathematics: complex dynamics, Kleinian groups, Teichmüller theory, moduli spaces, geometry... Starting using quasiconformal mappings in rational dynamics on the Riemann sphere since 1980-th had immediately led to the famous Sullivan's No Wandering Domain Theorem, which was a fundamental breakthrough in the theory. The key point is that in appropriate situations, quasiconformal mappings allow to construct a lot of different rational functions with required dynamics.

We discuss basics of quasiconformal mapping theory, with a proof of the main theorem (measurable Riemann mapping theorem). Then we pass to applications to dynamics of rational mappings of the Riemann sphere and Kleinian groups: finitely-generated discrete groups of conformal automorphisms of the Riemann sphere. We discuss the No Wandering Domain Theorem and structural stability in rational dynamics, and the analogous Ahlfors' Finiteness Theorem in Kleinian groups. And then Teichmüller theory, which deals with the space of different complex structures on a topologically marked closed surface.

This cours will be, on one hand, a continuation of Vladlen Timorin's cours (autumn 2013). On the other hand, it will be independent and self-contained and can be understood by those who did not follow Timorin's cours.

Cours outline

1. Quasiconformal mappings: introduction and main theorem.
2. Proof in the smooth case on the torus.
3. Grötsch inequality and proof in general case.
4. Application to rational dynamics: No Wandering Domain Theorem.
5. Structural stability of rational maps and holomorphic motions.
6. Kleinian groups. Ahflors' Finiteness Theorem and Measure Conjecture.
7. Teichmüller spaces.

Prerequisites: Complex analysis in one variable.

Textbooks:

L.Ahlfors. Lectures on quasiconformal mappings.

J.Hubbard. Teichmüller Theory and Applications to Geometry, Topology, and Dynamics.

C. McMullen. Riemann surfaces, dynamics and geometry. Course notes: http://www.math.harvard.edu/ ctm/home/text/class/harvard/275/09/html/base/rs/rs.pdf