Themes for курсовая работа 2013-2014

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Combinatorics and Representation theory (1-2 курс)

Young diagrams and tableaux are one of the most important and ubiquitous objects in combinatorics. They are nicely connected with representation theory of general linear groups, symmetric groups and certain analogue of them. They are also connected with symmetric polynomials directly and through the representation theory. In this курсовая работа, we study the combinatorics of Young tableaux and representation theory related with them. We may also consider generalization of Young tableaux and representation theory which were studied relatively recent: crystals of quantum groups, Fock spaces, Boson-Fermion correspondence, etc.

References

W. Fulton, Young Tableaux, London Math. Soc. Student Text **35**.
У. Фултон, Таблицы Юнга, Издательство МЦНМО.
J. Hong, S. Kang, Introduction of Quantum Groups and Crystal Basis, AMS.
Мива, Джимбо и Датэ, Солитоны, Издательство МЦНМО.

Application of Groebner basis in *Mathematica* (1-2 курс)

Groebner basis is generators of an ideal in a polynomial ring, which has a nice property to be calculated algorithmically. It has many applications to calculate algebraic structures around polynomial rings. One of the most popular symbolic computation program Wolfram *Mathematica* can compute Groebner basis of ideals by a command `GroebnerBasis[...]`, but utility commands which compute the algebraic structures around polynomial rings by applying the theory of Groebner basis, are not implemented (as I know). In this курсовая работа, we study the algorithms of the applications of Groebner bases and try implementing them by our own hands. Students who work on this курсовая работа are strongly recommended to attend спецкурс Groebner basis theory.

References

Cox, Little and O'Shea, Ideals, Varieties and Algorithms 3rd Ed., Springer.
Cox, Little and O'Shea, Using Algebraic Geometry 2nd Ed., Springer.

Representation Theory of $\mathfrak{sl}_2(\mathbb{C})$ (2-3 курс)

The Lie algebra $\mathfrak{sl}_2(\mathbb{C})$ is the simplest simple Lie algebra. Finite dimensional representations of $\mathfrak{sl}_2(\mathbb{C})$ is one of the most well-known results in the area of representation theory. In this курсовая работа, we study the representation theory of $\mathfrak{sl}_2(\mathbb{C})$, not only the finite dimensional representations, but also infinite dimensional representations. The main object is the BGG category, a category of "highest weight representations". This category is not semisimple category, and its representation theory is much more difficult than the representation theory of finite dimensional representations. We may also consider weight modules which are not of highest weight and representations in positive characteristic.

References

V. Mazorchuk, Lectures on $\mathfrak{sl}_2(\mathbb{C})$-modules, Imperial College Press.