

Projects for Students

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1. Continued fractions: Liouville's Theorem; Pell's equation; Hurwitz Theorem; Markov numbers. See first of all Courant and Robbins "What is Mathematics?", then also Rosen "Elementary Number Theory", or some suitable Russian-language alternative.
2. The Schwarzian derivative -- as
 - (i) a differential invariant,
 - (ii) the infinitesimal cross-ratio,
 - (iii) a cocycle for transformations of a differential operator under changes of the dependent variable.Curves in projective space, the Virasoro algebra, Poisson structure, differential invariants are all possible themes to develop. The basic reference is expected to be Ovsienko and Tabachnikov "Projective Differential Geometry Old and New".
3. Poisson Lie groups. Basic definitions involve learning about Poisson structure and Poisson actions. Learn about the r-matrix, the Drinfeld/Heisenberg double, Lie bi-algebras; learn about Poisson reduction and look at some examples. The main reference is expected to be the book "Integrable Systems" by Reyman and Semenov-Tian-Shansky.
4. Study of stability - especially in integrable systems. Several important "classical examples" include soliton solutions of KdV, NLS etc, rigid body. A recent result of Izosimov shows how to use bi-hamiltonian structure to analyse stability for an integrable system. The stability analysis of the Kovalewski top by means of Izosimov's method is expected to be an interesting and accessible problem.