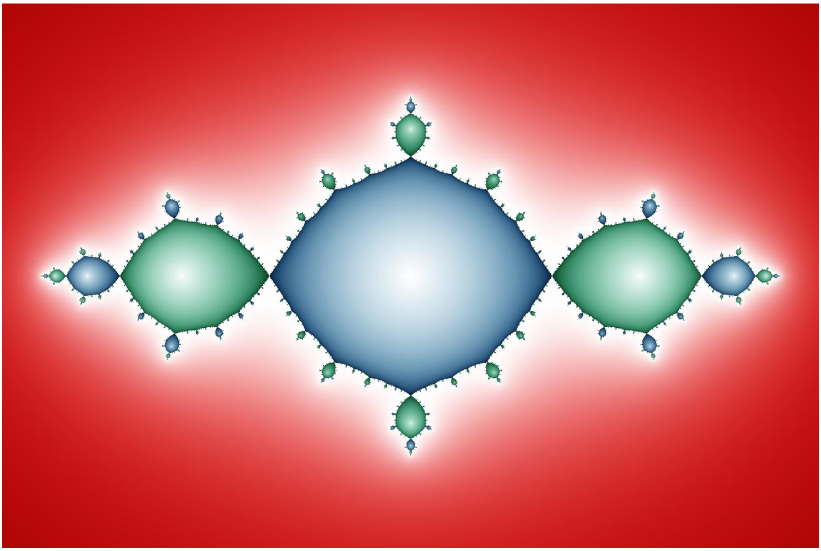
International Conference

Topological and Geometric Methods in Low-dimensional Dynamical Systems

May 11–16, 2014

Book of Abstracts



NRU HSE Moscow, 2014

The conference is supported by NRU HSE, AG Laboratory NRU HSE, MESRF grant, ag. 11.G34.31.0023, RFBR grant 14-01-06027-Г, Dynasty foundation grant 08/14

Organizing committee:

Ross Ptacek (Higher School of Economics)
Maria Sorokina (Baltic Institute of Mathematics)
Jerzy Smit (Baltic Institute of Mathematics)
Vladlen Timorin (Higher School of Economics)

Sponsoring institutions:

Laboratory of Algebraic Geometry and its Applications, Higher School of Economics
Independent University of Moscow
A*MIDEX Foundation, Aix-Marseille University
Dynasty Foundation
Russian Foundation for Basic Research
J.-V. Poncelet French-Russian Mathematics Laboratory

Schedule

Monday May 12

9:30 – 9:55	**Registration**
10:00 – 11:20	Pablo Aguirre
11:20 – 11:40	**Coffee break**
11:40 – 13:00	Katsutoshi Shinohara
13:00 – 15:00	**Lunch**
15:00 – 16:20	Dierk Schleicher
16:20 – 16:40	**Coffee break**
16:40 – 18:00	Alexey Klimenko

Tuesday May 13

10:00 – 11:20	Lex Oversteegen
11:00 – 11:20	**Coffee break**
11:20 – 12:20	Dzmitry Dudko
13:00 – 15:00	**Lunch**
15:00 – 16:00	Yulij Ilyashenko
16:00 – 16:20	**Coffee break**
16:10 – 16:35	Sergey Kryzhevich
18:00 –	**Conference Dinner**

Wednesday May14

10:00 – 11:20	Nikita Selinger
11:20 – 11:40	**Coffee break**
11:40 – 13:00	Hiroyuki Inou
13:00 – 15:00	**Lunch**
15:00 –	Excursion

Thursday May 15

10:00 – 11:20	Mary Rees
11:20 – 11:40	**Coffee break**
11:40 – 13:00	Dmitry Filimonov
13:00 – 15:00	**Lunch**
15:00 – 16:20	Sabyasachi Mukherjee
16:20 – 16:40	**Coffee break**
16:40 – 18:00	Alexey Glutsyuk

Friday May 16

10:00 – 11:20	Adam Epstein
11:20 – 11:40	**Coffee break**
11:40 – 13:00	Polina Vytnova
13:00 – 15:00	**Lunch**

Abstracts

Bifurcations of global invariant manifolds

Pablo Aguirre

We consider a codimension-two non-central saddle-node homoclinic point (SNH) in a model of a laser with optical injection. The aim of this talk is to understand how the overall dynamics changes qualitatively according to the configurations of the relevant global two-dimensional invariant manifolds. We compute the global invariant manifolds for representative parameter values to unravel how they organize the phase space. The results of our investigation provide a geometric explanation of how the two-dimensional manifolds rearrange themselves as global objects near the SNH point, how they are created/destroyed at the saddle-node transitions, and how the basins of the attracting objects of the model change in the process.

Expanding Thurston maps and matings

Dzmitry Dudko

We show that if a Thurston map f has no Levy obstruction and f is not covered by a torus endomorphism, then f is isotopic to an expanding (away from periodic critical points) map. As an application, we show that the geometric mating of two hyperbolic polynomials is a (topological) dynamical system on a 2-sphere if and only if there is no periodic ray connection.

Transversality principles in holomorphic dynamics

Adam Epstein

The moduli space of all degree D rational maps is an orbifold of dimension $2D-2$. We present a language for describing dynamically natural subspaces, for example, the loci of maps having

- specified critical orbit relations,
- cycles of specified period and multiplier,
- parabolic cycles of specified degeneracy and index,
- Herman ring cycles of specified rotation number, or some combination thereof.

We present a methodology for proving the smoothness and transversality of such loci. The natural setting for the discussion is a family of deformation spaces arising functorially from first principles in Teichmüller theory. Transversality flows from an infinitesimal rigidity principle (following Thurston), in the corresponding variational theory viewed cohomologically (following Kodaira-Spencer). Results for deformation spaces may then be transferred to moduli space. Moreover, the deformation space formalism and associated transversality principles apply more generally to finite type transcendental maps.

One-end finitely presented groups acting on the circle

Dmitry Filimonov
Joint work with Victor Kleptsyn

The present work belongs to a series of papers devoted to a further understanding of finitely generated groups acting on the circle. One of the general goals is a well-known question stated in 1980's by Sullivan and Ghys: is it true that if $G \subseteq Diff^2(S^1)$ is a finitely generated group of circle diffeomorphisms and its action is minimal then it is Lebesgue-ergodic? This is still a conjecture though there are many significant results. In one of the last papers Deroin, Kleptsyn and Navas showed that if G posesses the so-called property (*), than this allows to make distortion-controlled expansion and it is thus sufficient to conclude that the action is Lebesgue-ergodic. In this work we study possible one-end finitely presented subgroups of $Diff_+^{\gamma}(S^1)$, acting without finite orbits. Our main result establishes that any such action possesses the property (*).

On periodic orbits in complex billiards

Alexey Glutsyuk

A conjecture of Victor Ivrii (1980) says that in every billiard with smooth boundary the set of periodic orbits has measure zero. This conjecture is closely related to spectral theory. Its particular case for triangular orbits was proved by M. Rychlik (1989), Ya. Vorobets (1994) and other mathematicians, and for quadrilateral orbits in our recent joint work with Yu. Kudryashov. We present a new approach to planar Ivrii's conjecture: to study its complexified version with reflection to holomorphic curves (which is false in general) and to classify the counterexamples. We will show that the only "nontrivial" counterexamples with four reflections are formed by couples of confocal conics. If the time allows, we will discuss a small result concerning odd number of reflections and applications of these results to real billiards, including Plakhov's Invisibility Conjecture and Tabachnikov's commuting billiard problem.

New Phragmen-Lindelef theorems for functional cochains

Yulij S. Ilyashenko

Phragmen-Lindelef theorems hold not only for holomorphic functions, but rather for functional cochains. This property is a key point in the proof of the finiteness theorem for limit cycles. In its previous form (1991), the Phragmen-Lindelef theorems for functional cochains was proved only for the cochains that occur in the study of mohodromy mappings of polycycles. We present a general theorem for cochains not necessary related to the monodromy mappings. It looks more transparent and implies the previous one.

Straightening maps and similarity of parameter spaces in complex dynamics

Hiroyuki Inou

We give an overview of the results on similarity of parameter spaces in dynamics of complex polynomials in one variable. It is well-known that the Mandelbrot set has self-similarity. The natural homeomorphisms between baby Mandelbrot sets and the Mandelbrot set are simplest example of straightening maps. We formulate straightening maps and discuss if basic properties hold in other (higher-dimensional) parameter spaces. We also show that a straightening map for the anti-holomorphic quadratic family is not continuous at a specific parameter.

Arnold tongues for Josephson effect model equation

Alexey Klimenko
Joint work with Olga Romaskevich

We consider equation

$$\frac{dx}{dt} = \frac{\cos x + a + b\cos t}{\mu}, \qquad (t,x) \in \mathbb{R}^2/2\pi\mathbb{Z}^2 = \mathbb{T}^2, \quad (1)$$

which arises as a model for a Josephson junction (a "poor" contact of superconductors) in an external oscillating electromagnetic field. Here a and b depend on parameters of external field and current applied to the junction, and μ is a characteristic of the junction, so it is natural to let $\mu = \text{const}$ and vary a and b. Mathematically, dependence between current and voltage is expressed in terms of the rotation number of equation (1).

Recall that *Arnold tongues* are level sets $E_{\rho_0} = \{(a,b) : \rho(a,b) = \rho_0\}$ of the rotation number on the plane of parameters (a,b) that have nonempty interior.

The system of Arnold tongues for the family of equations was extensively studied in recent years. This research was inspired both by practical significance of the system and by its unusual properties from mathematical point of view.

While for a generic family of vector fields Arnold tongues exist for all $\rho_0 \in \mathbb{Q}$, in the family (1) they exist only for $\rho_0 \in \mathbb{Z}$. The reason is that in the coordinate $u = \tan(x/2)$ the flow map $P_{a,b}$ over the period is Möbius, but Möbius map has periodic points of period > 1 iff it is periodic itself.

Moreover, there is a symmetry $(x,t) \to (-x,-t)$ of equation (1), hence $P_{a,b}$ has a symmetry $P_{a,b}(x) = -P_{a,b}^{-1}(-x)$.

This means that inside tongues $P_{a,b}$ has two fixpoints $\pm z$, and on the tongue boundary these points coincide either at 0 or at π. Let $a_{0,k}(b)$ and $a_{\pi,k}(b)$ denote these values of a on the boundary of E_k. It can be shown that monotonicity in a yields that $a_{\dots,k}(b)$ are well-defined for every $b \geq 0$.

The survey of main results on this system will presented in the talk.

In particulal, we present our result (obtained jointly with Olga Romaskevich [1]) on asymptotics of the tongue boundaries $a_{\dots,k}(b)$ as $b \to +\infty$. It appears that they are asymptotically equivalent to the Bessel functions (after appropriate shift and scaling).

Theorem 1. *There exist positive constants* $C_1', C_2', K_1', K_2', K_3'$ *such that the following holds.*

For the parameters b, μ *and a number* $k \in \mathbb{Z}$ *satisfying inequalities*

$$|k\mu| + 1 \leq C_1'\sqrt{b\mu}, \qquad b \geq C_2'\mu$$

the following estimates hold

$$\left|\frac{a_{0,k}(b)}{\mu} - k + \frac{1}{\mu}J_k\left(-\frac{b}{\mu}\right)\right| \leq \frac{1}{b}\left(K_1' + \frac{K_2'}{\mu^3} + K_3' \ln\left(\frac{b}{\mu}\right)\right),$$
$$\left|\frac{a_{\pi,k}(b)}{\mu} - k - \frac{1}{\mu}J_k\left(-\frac{b}{\mu}\right)\right| \leq \frac{1}{b}\left(K_1' + \frac{K_2'}{\mu^3} + K_3' \ln\left(\frac{b}{\mu}\right)\right),$$

where $J_k(t)$ *is the Bessel function of the first kind.*

In particular, this means that each tongue has infinitely many points of zero width, which is also different from the behaviour of generic family.

Bibliography

[1] A. Klimenko, O. Romaskevich. Asymptotic properties of Arnold tongues and Josephson effect. arXiv:1305.6746. To appear in Moscow Mathematical Journal, 2014.

Turbulence in systems with dry friction

Sergey Kryzhebich
Joint work with N. Begun

We study a simple mathematical model of systems with dry friction. This is a particular case of so-called Filippov systems, that is ordinary differential equations with piecewise smooth right hand sides. Points of discontinuity form a finite union of piecewise smooth manifolds. Solutions of Filippov systems are in general non-unique. So-called sliding motions take place on subsets of these discontinuity manifolds.

It is shown that, Poincare mapping for the considered Filippov system may be reduced to a discontinuous map of a segment. We study properties of that 1D mapping, find its periodic points, superstable fixed points and chaotic invariant sets.

Non-landing parameter rays of the tricorn

Sabyasachi Mukherjee

It is well-known that every rational parameter ray of the Mandelbrot set lands at a single parameter. This is related to the fact that the parabolic parameters with given combinatorics are isolated in the Mandelbrot set. In this talk, we will consider the dynamics of anti-holomorphic quadratic polynomials and its connectedness locus, known as the tricorn. The topology of the tricorn differs vastly from that of the Mandelbrot set; for example, the tricorn contains parabolic arcs, which are real-analytic arcs consisting of quasi-conformally equivalent parabolic parameters. Our main theorem shows that certain parameter rays of the tricorn do not land; they accumulate on sub-arcs (of positive length) of these parabolic arcs. Time permitting, we will outline possible generalizations of our results to higher degree polynomial families, e.g. to the stretching rays of real cubic polynomials.

Combinatorial Models for Spaces of cubic polynomials

Lex Oversteegen
Joint work with A. Blokh, R. Ptacek, and V. Timorin

Thurston constructed a combinatorial model for the Mandelbrot set. No combinatorial model is known for the analogous spaces $\mathcal{M}_d$ of polynomials of degree $d \geq 3$. To address this problem we define *linked* geolaminations $\mathcal{L}_1$ and $\mathcal{L}_2$ (with critical sets divided into groups of specifically linked sets) and an *accordion*, i.e. the union of a leaf ℓ of $\mathcal{L}_1$ and leaves of $\mathcal{L}_2$ crossing ℓ. We show that an accordion behaves like a gap of one lamination and use this to prove that the maximal *perfect* (i.e., without isolated leaves) sublaminations of $\mathcal{L}_1$ and $\mathcal{L}_2$ coincide.

In the cubic case let $\mathcal{D}_3 \subset \mathcal{M}_3$ be the set of all cubic *dendritic* (i.e. having only repelling cycles) polynomials. Let $\mathcal{MD}_3$ be the space of all *marked* polynomials (P, c, w) where $P \in \mathcal{D}_3$ and c, w are critical points of P (perhaps, $c = w$). Let c^* be the *co-critical point* of c, i.e. the point $c^* \neq c$ with $P(c^*) = P(c)$ (if $c \neq w$) or c (if $c = w$). By Kiwi, to $P \in \mathcal{D}_3$ one associates its lamination $\sim_P$ so that each $x \in J(P)$ corresponds to a convex polygon G_x with vertices in $\mathbb{S}$. We associate to $(P, c, w) \in \mathcal{MD}_3$ its *mixed tag* $\theta(P, c, w)$ defined as $G_{c^*} \times G_{P(w)} \subset \overline{\mathbb{D}} \times \overline{\mathbb{D}}$ and show that mixed tags of distinct marked polynomials from $\mathcal{MD}_3$ are either disjoint or coincide. Let $\theta(\mathcal{MD}_3)^+ = \bigcup_{\mathcal{MD}_3} \theta(P, c, w) \subset \overline{\mathbb{D}} \times \overline{\mathbb{D}}$. The sets $\theta(P, c, w)$ form a partition of $\theta(\mathcal{MD}_3)^+$ and generate the corresponding quotient space of $\theta(\mathcal{MD}_3)^+$ denoted by MT_3. We prove that $\theta : \mathcal{MD}_3 \to \mathrm{MT}_3$ is continuous and thus MT_3 can serve as a combinatorial model space for $\mathcal{MD}_3$.

Persistent Markov partitions for rational maps

Mary Rees

Since its invention some thirty years ago, the Yoccoz puzzle has been a very important tool in results concerning local connectivity properties of Julia sets and the Mandelbrot set, for example. The Yoccoz puzzle is an example of a Markov partition. The use of Markov partitions is important in dynamics in general. In complex dynamics, parapuzzles, of which the Yoccoz parapuzzle is the prime example, have proved very useful in other contexts, with their use developed by Pascale Roesch, in particular, and by Aspenberg and Yampolsky, and Timorin, among others. This talk will start with a discussion of the properties which can be obtained for Markov partitions on some open subsets in parameter spaces of rational maps, which are not possible in general dynamics. A principal example is a neighbourhood of a geometrically finite rational map which might have parabolic periodic points. Then I will talk about the associated parapuzzle over the region of parameter space where the Markov partition persists, give a partial description, of the sets in the parapuzzle at each level, and raise some open questions.

Core Entropy for Quadratic Polynomials

Dierk Schleicher

In recent years, Bill Thurston investigated the "core entropy" of postcritically finite polynomials and asked various questions about its properties. In particular, he had a bet with John Hubbard whether the entropy depends continuously on external parameters.

We outline a proof of continuity for quadratic polynomials (joint with Dima Dudko) and relate it to the "biaccessibility dimension" for polynomial Julia sets (joint work with Philipp Meerkamp, as well as with Henk Bruin).

Classification of Thurston maps with parabolic orbifolds

Nikita Selinger
Joint work with M. Yampolsky

We give a classification of Thurston maps with parabolic orbifolds based on our previous results on characterization of canonical Thurston obstructions. The obtained results yield a partial solution to the problem of algorithmically checking combinatorial equivalence of two Thurston maps.

An example of the failure of C^1 Pesin theory

Katsutoshi Shinohara
Joint work with C. Bonatti and S. Crovisier

Let us consider a discrete dynamical system generated by a diffeomorphism on a compact smooth manifold. An invariant ergodic measure is called hyperbolic if none of its Lyapunov exponents is equal to zero. If the diffeomoprhism is of regularity C^r where r is greater than or equal to two, then the famous Pesin's theorem tells us that for generic points of the measure we can find the stable/unstable manifold passing through them. One may fancy if the result is true for C^1 diffeomorphisms. In this talk, I will give an example of C^1 systems where the Pesin theory is not true.

Kinematic fast dynamo problem for planar maps

Polina Vytnova

Dynamo theory studies the mechanism of generation of magnetic fields in electrically conducting fluids such as liquid core of the Earth or atmospheres of stars. The classical kinematic fast dynamo problem concerns the evolution of a magnetic field in a conducting fluid flow in the presence of small diffusion. This question has a discrete analogue, so called kinematic fast dynamo problem for maps.

Let w_ε be the d-dimensional Gaussian density with isotropic variance ε. Given a diffeomorphism $P\colon \mathbb{R}^d \to \mathbb{R}^d$ we define an operator on vector fields $T_\varepsilon\colon v \mapsto w_\varepsilon * P_* v$, where $*$ stands for convolution and $P_* v(z) = \mathrm{d}P(P^{-1}z)v(P^{-1}z)$ is an induced operator on vector fields associated to P. Do there exist a vector field v_0 with a compact support $M\colon \stackrel{\text{def}}{=} \operatorname{supp} v_0$ and a C^k volume preserving diffeomorphism P of $\mathbb{R}^d$, such that all partial derivatives are bounded $\sup\limits_{z\in\mathbb{R}^d\setminus M} |\partial_i P_j(z)|$ for some small μ, such that the magnetic energy grows exponentially fast with number of iterations? In other words,

$$\lim_{\varepsilon\to 0}\lim_{n\to\infty}\frac{1}{n}\ln\int_{\mathbb{R}^d}|T_\varepsilon^n v_0(z)|\mathrm{d}z > 0, \tag{2}$$

where T_ε is connected to P via equation above.

We give a positive answer to this question; and the construction is very explicit. An advantage of the construction is that it can be turned, after a slight modification, into Poicaré map of a real 3-dimensional flow and become a dissipative fast dynamo in $\mathbb{R}^3$.

Book of Abstracts, Moscow 2014

HIGHER SCHOOL OF ECONOMICS
NATIONAL RESEARCH UNIVERSITY

Bałtycki Instytut Matematyki Warszawa

A*Midex
Initiative d'excellence Aix-Marseille

Династия

РФФИ

МЦНМО