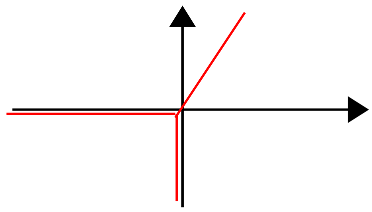
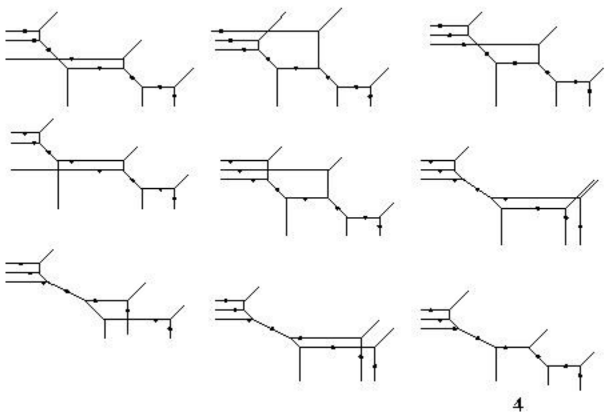
TROPICAL GEOMETRY (3^{rd}-4^{th} module)

Tropical numbers simulate the behavior of the degrees of polynomials with respect to summation and multiplication: the tropical product of a and b equals a+b, the tropical sum equals max(a,b) unless a=b, and, in the latter case, it can attain every value that does not exceed a. Together with the neutral element $-\infty$, the tropical numbers form a semifield. Here is, for instance, the graph of the function $y=x^2+1$, if all the symbols (“^{2}”, “+”, “1”, and even “=”) are understood in the tropical sense:



Mysteriously enough, algebraic geometry over this semifield (i. e. the geometry of zero loci of tropical polytopes) turns out to be very similar to the classical one: the same question in both geometries often has the same answer. This similarity is very useful for the classical algebraic geometry, because switching to the tropical one makes all questions much simpler: the tropical geometry is essentially the study of certain piecewise-linear objects (see the picture above).

For instance, the following question is not obvious at all in the classical algebraic geometry: how many rational cubic curves (i. e. cubic polynomial maps of a projective line to a projective plane) pass through eight given generic points in the projective plane? Mikhalkin's tropical correspondence theorem states (in a much more general setting) that the answer is the same as over the tropical semifiled. It is now a purely combinatorial question to count tropical rational cubic curves, passing through eight given points in a tropical plane, the answer is shown below and equals 12:



<u>Program</u>: tropical algebra, amoebas, tropical Bezout theorem, introduction to enumerative geometry, Mikhalkin's tropical correspondence theorem, sparse resultants and A-discriminants, secondary polytope, deformations of algebraic curves, proof of Mikhalkin's theorem for 1 and 2 nodes, outline for the general case.

<u>References</u>: I. Itenberg, G. Mikhalkin, E. Shustin. Tropical algebraic geometry. http://books.google.fr/books?id=4JP7ofjHVh8C
M. Kazarian. Tropical geometry. http://www.mccme.ru/dubna/2006/notes/Kazaryan.pdf

<u>Prerequisites</u>: The course can be taken by 2^{nd} year students and higher. The basic knowledge of toric varieties and Newton polytopes is required.