QUANTUM ALGEBRAS

This course is a НИС mainly for master course and 3rd and 4th years of undergraduate. The prerequisite knowledge is linear algebra and basic algebra. The knowledge of Lie groups and Lie algebras is not required but it will help you to understand. Note that this course is given in English.

In this seminar, we study and discuss representation theory of some superalgebras. Here "superalgebras" are associative algebras which consist of fermionic (odd) elements together with bosonic (even) elements. Usual algebras in Lie theory consist of only bosonic elements; namely defining relations between generators in those usual algebras are given by commutation relations of form $XY-YX=Z$. Universal enveloping algebras of Lie algebras and Weyl algebras (algebras of differential operators) are such algebras. On the other hand, it is natural and reasonable to consider algebras which has generators with defining relations given by anti-commutation relations like $XY+YX=Z$. Such generators are called "fermionic (odd)", and associative algebras which are generated by bosonic and fermionic elements are called superalgebras. They are natural generalization of universal enveloping algebras and Weyl algebras.

"Representation theory of superalgebras" consists of several topics; some of them are natural analogue of the fact from representation theory of usual algebras, and some of them are particular for superalgebras; moreover, some of them have a connection with mathematical physics and other areas of mathematics. The goal of the seminar is to study such topics on superalgebras according to our interests and to survey the "representation theory of superalgebras." The topics may includes:

- classification of finite-dimensional Lie superalgebras,
- representation theory of finite/Kac-Moody Lie superalgebras,
- representation theory of Clifford algebras,
- BRST cohomology and quantum Hamiltonian reduction,
- finite/affine W-algebras including their super-analogue.

Students are recommended to give a talk (presentation) on some topics in representation theory of superalgebras according to their own interests. Giving talks is NOT mandatory, but students who give a talk (presentation) in the seminar in each semester will pass and obtain grade according to their talk. Students who do not give a talk must take an exam at the end of semesters, and their grades depends on the result of the exam.

Toshiro Kuwabara
Takashi Takebe