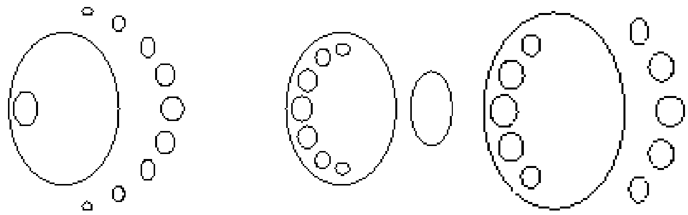
REAL ALGEBRAIC GEOMETRY (1st module)

The 16th of the 23 problems for the 20th century, published by David Hilbert in 1900, was devoted to the topology of smooth algebraic curves on the real projective plane. Everybody knows that every such curve of degree 2 is an ellipse, which is the boundary of a topological disc. An interesting problem for a 2nd year student is to prove that every curve of degree 4 consists of the boundaries of ≤ 4 disjoint discs or two nested discs. For the degree 6, even Hilbert himself was unable to complete the classification: he knew that every such curve consists of the boundaries of ≤ 11 discs, and, in the case of exactly 11 discs, at most the following three cases are possible:



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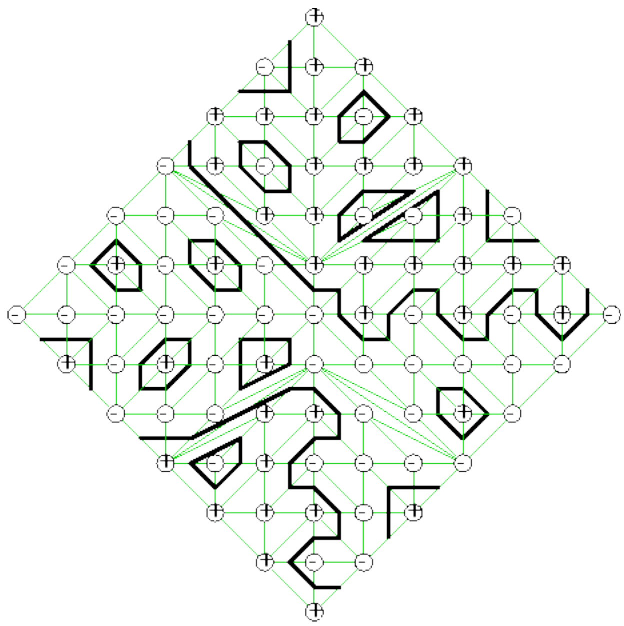
However, he did not know whether the 3rd of these cases takes place for any curve of degree 6. It was not until 1969 that Gudkov constructed an example of such a curve.

Hilbert's 16th problem was the starting point for modern real algebraic geometry (the study of geometry and topology of zero loci of real polynomials), and Gudkov's construction is a special case of one of the most important tools in this science: Viro's patchworking. This tool allows to construct polynomials with a prescribed topological type of the zero locus. For instance, here is the patchworking construction for Gudkov's curve:



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Program: Introduction to real algebraic geometry, Harnack's curve inequality, Viro's patchworking, moment maps, $\mathbb{R}_+$-toric varieties, proof of the patchworking theorem.

References: Viro's textbooks (http://www.pdmi.ras.ru/~olegviro/educ-texts.html)

Prerequisites: The course can be taken by 2nd year students and higher.