Alexey L. Gorodentsev

One semester course «Sheaves And Homological Algebra»

This is an introduction to the theory of sheaves and supplying homological algebra — commonly used technique for handling locally defined objects[1] on a manifold X. In algebraic and/or differential geometry and topology it allows to produce global geometric and topological invariants of X from those local data. In non-commutative geometry it gives various geometric style invariants of categories equipped with Grothendieck topologies.

Duration: September – December, 2015.

Recommended textbooks:

[D] *V. I. Danilov. Cohomology of Algebraic Manifolds.* English version in: Algebraic Geometry II: Cohomology of Algebraic Varieties. Algebraic Surfaces. Encyclopaedia of Mathematical Sciences. Book 35. Springer (1995). Russian version in: Алгебраическая геометрия – 2. Итоги науки и техники. Совр. пробл. математ. Фунд. напр. Т. 35. Москва, ВИНИТИ, 1989, с. 5 – 130.

[GH] *P. Griffiths, J. Harris. Principles of Algebraic Geometry.*

[GM] *S. I. Gelfand, Yu. I. Manin. Methods of Homological Algebra. Part I.*

[I] *B. Iversen. Cohomology of Sheaves.*

[W] *C. A. Weibel. An Introduction to Homological Algebra.*

Prerequisites: the first 3 semesters (6 modules) of the standard courses «Algebra», «Calculus», and «Geometry/Topology» given at our faculty or at IUM.

Preliminary program:

1. Categories, functors, pre-sheaves. The main working examples: open sets of a topology and simplicial sets. Cateory of functors, Yoneda's lemma. Adjoint functors. (Co) limits of diagrams. ([GM], [W])
2. Sheaves on topological spaces. Stalks and the étalé space of a sheaf. Sheafification. Pull back and push forward. Abelian sheaves. ([I], [GM])
3. Complexes and (co)homologies. Long exact sequence of cohomologies. The Koszul complexes. Cohomologies and filtered colimits. Spectral sequences of filtered complexes, bicomplexes, and exact couples. ([GM], [D], [W])
4. Global sections, flabby sheaves, and the Godement resolution. Sheaf cohomology and hypercohomology. Acyclic resolutions. The Mayer – Vietoris exact sequence and Čech's resolution. Acyclic coverings and Cartan's criterion for acyclicity. The Čech cohomologies. ([I], [D])
5. Fine and soft sheaves. The sheaves of differetial forms, the Poincaré lemma and the DeRham theorem. ([GM], [GH], [D])
6. Higher direct images. The Leray spectral sequence. ([I], [D], [GH])
7. (If the time allows.) Coherent sheaves in algebraic geometry: examples and applications. Acyclicity of affine varieties. Cohomologies of invertible sheaves on projective spaces. ([D])
8. (If the time allows.) Grothendieck topologies and sheaves on sites. ([GM])

[1] e.g. functions with restricted domains of definition, local sections of vector bundles, locally defined continuous mappings etc