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One semester course «Sheaves And Homological Algebra»

This is an introduction to the theory of sheaves and supplying homological algebra — commonly used technique for handling locally defined objects¹ on a manifold $X$. In algebraic and/or differential geometry and topology it allows to produce global geometric and topological invariants of $X$ from those local data. In non-commutative geometry it gives various geometric style invariants of categories equipped with Grothendieck topologies.

Duration: September – December, 2015.

Recommended textbooks:


Prerequisites: the first 3 semesters (6 modules) of the standard courses «Algebra», «Calculus», and «Geometry/Topology» given at our faculty or at IUM.

Preliminary program:

1. Categories, functors, pre-sheaves. The main working examples: open sets of a topology and simplicial sets. Category of functors, Yoneda’s lemma. Adjoint functors. (Co)limits of diagrams. ([GM], [W])

2. Sheaves on topological spaces. Stalks and the étalé space of a sheaf. Sheafification. Pull back and push forward. Abelian sheaves. ([I], [GM])

3. Complexes and (co)homologies. Long exact sequence of cohomologies. The Koszul complexes. Cohomologies and filtered colimits. Spectral sequences of filtered complexes, bicomplexes, and exact couples. ([GM], [D], [W])


5. Fine and soft sheaves. The sheaves of differential forms, the Poincaré lemma and the DeRham theorem. ([GM], [GH], [D])

6. Higher direct images. The Leray spectral sequence. ([I], [D], [GH])

7. (If the time allows.) Coherent sheaves in algebraic geometry: examples and applications. Acyclicity of affine varieties. Cohomologies of invertible sheaves on projective spaces. ([D])

8. (If the time allows.) Grothendieck topologies and sheaves on sites. ([GM])

¹e.g. functions with restricted domains of definition, local sections of vector bundles, locally defined continuous mappings etc