Topology 2: brief description and syllabus

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Singular homology and cohomology provide tractable but efficient invariants of topological spaces: these invariants are relatively easy to calculate but they can often be used to tell apart spaces which are not homeomorphic or not homotopy equivalent. In this course we will prove the main properties of the singular (co)homology theory and consider many examples and applications, including those which are relevant to fields other than topology (such as analysis, geometry, algebra, computer science). We will caver all material which is normally covered in a graduate introductory algebraic topology course.

The main prerequisites for this course are basic algebra (groups, rings, fields), linear algebra, and point-set topology e.g. as covered in Topology 1 (topological and metric spaces, CW-complexes, covering maps and the fundamental group). We will recall some or all of these if necessary.

- Introduction. The basic properties of singular homology. The Steenrod-Eilenberg axioms. First applications: Brower's fixed point theorem; the invariance of dimension of a topological manifold.
- The definition of singular homology via singular chains. Basic homological algebra: exact sequences, complexes, 5-lemma, chain homotopy. Applications: Jordan's theorem and the invariance of domain.
- Homotopy invariance and excision for singular homology. The acyclic model theorem. Further applications: the degree of a self-map of a sphere.
- Homology with coefficients and the universal coefficient theorem. The transfer map, the Borsuk-Ulam theorem and some of its applications.
- Cellular chain complexes; cellular decompositions of Grassmannian manifolds.
- The Künneth formula for homology.
- The basic properties of the cohomology groups (the Steenrod-Eilenberg axioms, cellular cochain complexes, the Künneth formula).
- The ring structure in cohomology. Examples of calculation and first applications (division algebra structures on Euclidean spaces, the Schwarz genus and the Lusternik Schnirelmann category etc.).
- The cap product and its basic properties.
- Topological manifolds and the Poincaré-Lefschetz duality. Applications.
- Poincaré duality and intersection theory. The class of the graph of a self-map of a smooth manifold and the Lefschetz fixed point theorem for manifolds. Some other versions of the fixed point theorem and applications.

Alternatively, if it turns out that everyone is familiar with these topics already, we could cover some of the more advanced ones, such as spectral sequences, characteristic classes or the topological K-theory.

The classes take place on Tuesday at 5pm – 8pm in room 207, the Mathematics department of the HSE Moscow. The lecturer's office hours are 3.30pm – 5pm Monday and Friday, room 1006, HSE math department.

There will be a class test sometime in late March/early April. The final exam, the test and homework count respectively 60%, 20% and 20% towards the final mark.

The main references are

- *A Course in Homotopy Theory* by D. Fuchs and A. Fomenko.
- *Algebraic Topology* by A. Hatcher, freely available online at http://www.math.cornell.edu/~hatcher/AT/ATpage.html.
- *Characteristic classes* by J. Milnor and J. Stasheff.

Occasionally we'll be using other sources as well.