Topology 1: brief description and syllabus

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Informally speaking, topology is a branch of mathematics that tries to answer the question whether two geometric shapes can be transformed into one another by stretching but without cutting or tearing. Quite surprisingly, these questions arise in almost every area of mathematics, from probability to algebra. So in particular, taking a graduate-level course in analysis or any kind of geometry requires some background in topology. On the other hand, topology in itself is a fascinating subject and it is is full of surprises. This course is intended as an introduction to topology. We will first cover some point-set topology. We will focus on the material which we'll need in the rest of the course and which is also likely to be a prerequisite for taking courses in other disciplines. Then we'll look at the basics of algebraic and geometric topology and consider some applications.

There are no prerequisites for this course apart from basic set theory and some algebra (basically, all we'll ever need is the notion of a group and a bit of linear algebra). However, if you've already taken a course in analysis or differential geometry, this would help. Here is a tentative syllabus.

- Basic definitions (topological spaces, continuous maps, compact spaces, separation axioms, quotient topology, homotopy between maps) and first applications (the main theorem of algebra).
- Metric spaces. The completion of a metric space; Banach's fixed point theorem. Compactness criteria for metric spaces. The Stone-Weierstrass theorem. The Hausdorff metric.
- The Euler characteristic and the classification of surfaces. Applications (the number of the ovals of a smooth real projective curve; regular polyhedra in 3-space, etc.)
- The Riemann-Hurwitz formula and some applications (the genus of a plane curve, the Hurwitz bounds on the order of the automorphism groups of complex curves).
- CW-complexes. Subcomplexes and quotient complexes. Homotopy extension property and applications.
- The fundamental group. Covering spaces and the correspondence between subgroups of the fundamental group, transitive π_1-sets and connected covering spaces of a given space. Applications to group theory.

If time allows, we'll also cover some homology theory and/or homotopy theory.

The classes will take place on Wednesday from 5pm till 8pm in room 304, the math department of the Higher School of Economics, Moscow. The lecturer's office hours are 3.30pm – 5pm Monday and Friday, room 1006, HSE Moscow, math department.

There will be a class test sometime in late October/early November. The final exam, the test and homework count respectively 60%, 20% and 20% towards the final mark.

The main references are

- *Algebraic Topology* by A. Hatcher, freely available online at `http://www.math.cornell.edu/~hatcher/AT/ATpage.html`.
- *A Basic Course in Algebraic Topology* by W.S. Massey, Springer GTM, 1991.
- *Topology I lecture notes* by V.V. Prasolov and A.B. Sossinsky, MCCME Publishers, 2007.

Occasionally we'll be using other sources as well.