Exercises 1. Euler–Lagrange equation.

Calc. of Var. and Op. Co., bachelor 3-4 year, **18.01.2016**

Deadline — Feburary 11, 2016.

1◇1 Solve the following Simplest problems of Variational Calculus:

(a) $J(x) = \int_0^1 (x + \dot{x})^2 dt, \quad x(0) = 0, \quad x(1) = 1;$

(b) $J(x) = \int_1^e \left[\frac{2x}{t} + x\dot{x} + t^2\dot{x}^2\right] dt, \quad x(1) = 1, \quad x(e) = 0.$

1◇2 (Weierstrass) Prove that the following extremal problem

$$\int_0^1 t^2\dot{x}^2\, dt \to extr, \qquad x(0) = 0, \quad x(1) = 1$$

does not has a solution.

1◇3 Solve the following problem with a free end:

$$J(x) = \int_0^2 [2tx + \dot{x}^2] dt, \qquad x(0) = 0.$$

1◇4 Solve the following problem without restrictions:

$$J(x) = \int_1^e \left[t\dot{x}^2 + \frac{x^2}{t} + \frac{2x\ln t}{t}\right] dt.$$

1◇5 Calculate the distance between the parabola $y = x^2$ and the line $y = x - 5$.

1◇6 The segment of the curve $x = x(t)$ with ends (a, A) and (b, B) rotates around axis Ox. What should be a curve to the surface area of the rotation was minimal?