Elliptic Functions

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- If there are errors in the problems, please fix *reasonably* and solve them.
- The rule of evaluation is:

  $$(\text{your final mark}) = \min\left\{\text{integer part of } \frac{3}{2}(\text{total points you get}), 10\right\}$$
- About twenty problems will be given till the end of the semester.
- This rule is subject to change and the latest rule applies.
- The deadline of **9** – **10**: 29 February 2016.

9. (1 pt.) Prove that when the modulus $k \in (0,1)$ tends to 1,

$$\begin{aligned} K(k) &\to \infty, \\ \operatorname{sn}(u,k) &\to \tanh u = \frac{\sinh u}{\cosh u}, \\ \operatorname{cn}(u,k),\ \operatorname{dn}(u,k) &\to \operatorname{sech} u = \frac{1}{\cosh u}. \end{aligned}$$

10. (1 pt.) Finish the computation omitted in the lecture and complete the proof of the addition formula of $\operatorname{sn} u$. Then, using it, prove the addition formulae of $\operatorname{cn} u$ and $\operatorname{dn} u$:

$$\begin{aligned} \operatorname{cn}(u+v) &= \frac{\operatorname{cn} u\ \operatorname{cn} v - \operatorname{sn} u\ \operatorname{sn} v\ \operatorname{dn} u\ \operatorname{dn} v}{1 - k^2 \operatorname{sn}^2 u\ \operatorname{sn}^2 v}, \\ \operatorname{dn}(u+v) &= \frac{\operatorname{dn} u\ \operatorname{dn} v - k^2 \operatorname{sn} u\ \operatorname{sn} v\ \operatorname{cn} u\ \operatorname{cn} v}{1 - k^2 \operatorname{sn}^2 u\ \operatorname{sn}^2 v}. \end{aligned}$$