

Elliptic Functions

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- If there are errors in the problems, please fix *reasonably* and solve them.
- The rule of evaluation is:

$$(\text{your final mark}) = \min \left\{ \text{integer part of } \frac{3}{2}(\text{total points you get}), 10 \right\}$$

- About twenty problems will be given till the end of the semester.
- This rule is subject to change and the latest rule applies.
- The deadline of **16 – 17**: 16 May 2016.

16. Check that the 1-form

$$\tilde{\omega}_3(P, Q) := \frac{1}{2} \left(\frac{w + w_1}{z - z_1} - \frac{w + w_2}{z - z_2} \right) \frac{dz}{w}$$

satisfies the properties required in the lecture (18 April 2016) in the following cases (i) and (ii):

(i) (1 pt.) $P = (z_1, w_1)$, $Q = (z_2, w_2)$ are not neither branch points nor ∞_{\pm} .

(Hint: You have to check the holomorphicity separately at (1) $(z, w) \in \mathbb{C}^2$, $z \notin \{z_1, z_2, \alpha_0, \dots, \alpha_3\}$; (2) $(z_i, -w_i)$ ($i = 1, 2$); (3) $(\alpha_j, 0)$; (4) ∞_{\pm} (use the coordinate $\xi = 1/z$).)

(ii) (1 pt.) At least one of P and Q is a branch point.

(iii) (1 pt.) Find $\tilde{\omega}_3(P, Q)$ with the same properties when $P = \infty_{\pm}$ or $Q = \infty_{\pm}$ or both of them are at infinity. (Hint: When $z_1 \rightarrow \infty$, $w_1 \sim \pm\sqrt{a}z_1^2$. Hence naive limit $\lim_{z_1 \rightarrow \infty} \tilde{\omega}_3(P, Z)$ diverges. Find an appropriate $\lambda = \lambda(z_1)$ and take the limit $\lim_{z_1 \rightarrow \infty} (\tilde{\omega}_3(P, Q) - \lambda\omega_1)$.)

17*. (2 pt.) Find meromorphic 1-forms as in **16** on the elliptic curve $\bar{\mathcal{R}}_{\varphi}$ when $\deg \varphi = 3$.