

Representations of affine and vertex operator algebras

The theory of Lie groups and Lie algebras is one of the central areas of modern mathematics. It has various interrelations with algebraic geometry, combinatorics, theory of symmetric functions, integrable systems, classical and quantum field theories. Lie groups and Lie algebras usually show up as the sets of symmetries of objects of a theory. For example, infinite-dimensional Lie algebras (such as affine Kac-Moody algebras) turn out to be very important for the description of many quantum field theories: namely, they are realized as symmetries of the spaces of states. Infinite-dimensional Lie algebras also play an important role in the theory of integrable systems and in algebraic geometry. It turns out that it is very natural in this context to consider more general algebraic objects, the vertex operator algebras. VOAs capture the main properties of the Lie algebras and have rich additional structure. Vertex operator algebras proved to be very useful in many situations; the classical example is the KP integrable hierarchy. They are also extensively used in modern algebraic geometry.

Our goal is to give an introduction to the theory of infinite-dimensional Lie algebras and vertex operator algebras. We describe the main definitions, constructions and applications of the theory. The course is aimed at PhD students and master students. Prerequisites: basic Lie theory.

Preliminary program:

1. Heisenberg algebras and Fock modules.
2. Virasoro algebras, Verma modules.
3. Infinite matrices.
4. The Lie algebra $\widehat{\mathfrak{sl}}_2$, representations and theta-functions.
5. Boson-fermion correspondence, Schur polynomials, KP hierarchy.
6. Affine Kac-Moody Lie algebras: structure theory.
7. Affine Kac-Moody Lie algebras: representation theory.
8. Vertex operator algebras and Lie algebras.
9. Associativity and operator product expansion.
10. Representation theory of vertex operator algebras.

Literature.

1. Kac, V. Infinite dimensional Lie algebras, Cambridge University Press (1994).
2. Frenkel E., Ben-Zvi D. Vertex algebras and algebraic curves (AMS, 2001)
3. Kac V., Raina A. Bombay lectures on Highest weight representations of infinite dimensional Lie algebras (WS, 1987)
4. Kac V. Vertex algebras for beginners (AMS, 1997)