

**Representations of affine and vertex operator algebras**  
**Homework 2**

1. Prove the equality  $\exp(a\partial/\partial x)\exp(bx) = \exp(ab)\exp(bx)\exp(a\partial/\partial x)$ .
2. Let  $x_j = \frac{\varepsilon_1^j + \dots + \varepsilon_N^j}{j}$ . Prove that  $S_k(x_1, x_2, \dots)$  is equal to the trace of the matrix  $\text{diag}(\varepsilon_1, \dots, \varepsilon_N)$  in the  $GL_N(\mathbb{C})$  module  $S_k(\mathbb{C}^N)$ .
3. Solve problem 2 for a general irreducible highest weight  $GL_N$  module.
4. Prove that all finite-dimensional representations of the Witt algebra are trivial.
5. A vector  $v$  in a representation of the Virasoro algebra is called singular if  $L_n v = 0$  for all positive  $n$ . Find all singular vectors  $v$  in the Verma module  $M_{c,h}$  such that: a).  $L_0 v = (h + 1)v$ , b).  $L_0 v = (h + 2)v$ , c).  $L_0 v = (h + 3)v$ .