PDE-17, recitation 1, Ordinary and Partial Differential Equations

1. Find all the eigenvectors and eigenvalues of the Laplace operator on a circle.

2. Prove that the Laplace operator on a circle is self-adjoint in L_2.

3. Prove that the Laplace operator on a circle is non-positively defined.

4. Find the general solution of the Cauchy problem for the heat equation in the space of the trigonometric polynomials on a circle.

5. The same for the wave equation.

6. The same for the Laplace equation.

Solve the following Cauchy problems on a circle $S^1 = \mathbb{R}/2\pi Z$:

7. $u_t = u_{x^2},\ u|_{t=0} = \sin^2 x$

8. $u_{t^2} = u_{x^2},\ u|_{t=0} = \sin^2 x, \quad u_t|_{t=0} = 0.$

9. $u_{t^2} = -u_{x^2},\ u|_{t=0} = \sin^2 x, \quad u_t|_{t=0} = 0.$