

Announcement of the course “Partial differential equations”, Spring 2017, Instructor Yu. Ilyashenko

Development of the theory of partial differential equations (PDE) may be split to three periods when the theory made use of:

- classical Analysis
- Functional Analysis
- Theory of Distributions

All the three periods will be illustrated by the theory of the first order equations, and the three main equations of the second order: Laplace, Heat and Wave equations. Elements of the general theory (that forms a minor part of the whole PDE theory) will be delivered also.

PROGRAM

1. First order equations: linear, quasilinear and nonlinear (Arnold’s approach)
2. Normal form of the differential operators of the second order
3. Laplace operator and harmonic functions
4. Cauchy problem for the Heat equation
5. Boundary value problem for the Heat equation. Separation of variables and Fourier series
6. String equation and D’Alembert formula
7. Oscillations of the bounded string: D’Alembert and Fourier methods
8. Maxwell equations and electromagnetic origine of light
9. Cauchy problem for the Wave equation
10. Sturm-Liouville equation
11. Distributions and operations on them
12. Fundamental solutions and their use for nonhomogeneous equations
13. Sobolev spaces
14. Solution of the Dirichlet problem for the Laplace equation
15. Kovalevski theorem
16. Well and ill posed problems

17. Spectrum of the Laplace operator

18. Variational methods in the estimate of the eigenvalues of the Laplace operator

Prerequisites: Analysis, 4 semesters, including Fourier transform and measure theory, Linear Algebra and elements of Functional Analysis: Hilbert spaces and integral operators. Some necessary properties of these operators will be delivered in the course.

Bibliography:

1. V. Arnold, Partial differential equations
2. V. Vladimirov, Equations of mathematical physics
3. O. Oleynik, Lectures on partial differential equations
4. M. Shubin, Lectures on equations of mathematical physics
5. Problems on partial differential equations, A. Shamaev editor