

PDE-17, recitation 2. First integrals. Linear partial differential equations

First integrals

1. Let a linear autonomous system on the plane be of the type a) saddle b) node c) focus d) center. Does there exist a non-constant continuous first integral of this system in some neighborhood of zero?
2. a. Find all the continuous first integrals for a system

$$\dot{x} = Ax, A = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}.$$

in a neighborhood of zero.

- b. For which λ the system

$$\dot{x} = (A - \lambda E)x$$

has continuous first integrals in some neighborhood of zero?

3. Does there exist a continuous first integral of a system

$$\dot{x} = x^2 - 1, \dot{y} = 1 - y^2,$$

defined a) in the unite circle $x^2 + y^2 \leq 1$?

b) In the whole plane?

Linear equations

4. Find a general solution of the equation $yu_x + (x - x^3)u_y = 0$.
5. Find a general solution of the equation $yu_x + (x^3 - x)u_y = 0$.
6. Is it correct that the following Cauchy problem: $yu_x + (x^3 - x)u_y = 0, u|_{x=0} = \sin y$ has a unique solution in some neighborhood of zero?
7. Is the following Cauchy problem $yu_x + (x^3 - x)u_y = 0, u|_{x=0} = \sin y$ solvable in a neighborhood of zero?

8. Is the following Cauchy problem $yu_x + (x - x^3)u_y = 0$, $u|_{x=0} = \cos y$ solvable in a neighborhood of zero? If yes, is the solution unique in some neighborhood of the initial line? Find the maximal domain in which the solution exists and is unique.

Solved in the class: 1, 2a, 4..

HW: 2b, 3a, b, 5, 6t, 7, 8.