Recitation and HW 5. Coordinate changes in the second order differential operators

1. For what α the problem $(Du)^2 = 1,\ u|_{x=0} = \alpha x$ has a solution?
2. Prove that the Laplace operator is preserved by those and only those linear transformations that are orthogonal.
3. Prove that the Laplace operator in the plane is preserved by those and only those diffeomorphisms that are linear and orthogonal.
4. Prove that the Helmholtz operator is preserved by those and only those linear transformations that preserve the Minkovski metric.
5. Prove that the Helmholtz operator in the plane is preserved by those and only those linear transformations that have the form

$$\begin{pmatrix} \operatorname{ch} t & \operatorname{sh} t \\ \operatorname{sh} t & \operatorname{ch} t \end{pmatrix} \tag{1}$$

6. Prove that the Helmholtz operator in the plane is preserved by those and only those diffeomorphisms that are linear and have the form (1).
7. Bring the equation $u_{tt} - a^2 u_{xx} = 0$ to the canonical form.
8. Find the general solution of the previous equation.
9. Shubin 1-1a.
10. Shubin 1-1b.
11. Shamaev 2.7, all the three parts,

Solved in the class: 2, 4, 5, 7.

HW: 1, 3, 6, 8 – 11.