Recitation 4. Symplectic structures and non-linear equations

Liner algebra digression

1. A skew scalar product in any basis is given by a skew-symmetric matrix
2. A skew symmetric matrix of order two is equivalent to $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
3. A non-degenerate skew product is equivalent to $dp_1 \wedge d_1 + \cdots + dp_n \wedge dq_n = dp \wedge dq$ in multiple notation
4. $dp \wedge dq(\xi, \eta) = \sum_1^n A_j$, where A_j is the area of the projection of the parallelogram spanned by ξ, η to the $(\frac{\partial}{\partial p_j}, \frac{\partial}{\partial q_j})$ plane.
5. A non-degenerate skew scalar product exists in an even-dimensional space only.
6. If all the vectors of a subspace of a symplectic $2n$-space are mutually skew orthogonal then the dimension of this subspace is no greater than n.

   Non-linear PDE of the first order
7. Derive the characteristic equation for a linear first order PDE from the general characteristic equation.
8. Same for quasilinear PDE
9. Investigate existence and uniqueness of a solution of a problem

   $$(Du)^2 := (u_x)^2 + (u_y)^2 = 1 \qquad (1)$$

   with the initial condition

   $$u|_{y=x^2} = 0$$

   in the domains a) $y < x^2$, b) $y > x^2$.
10. Find the curve that bounds the domain of the sulution in Problem a).
11. The same for equation (**??**) with the initial condition $u|_\gamma = 0,\ \gamma = (\frac{x}{a})^2 + (\frac{y}{b})^2 = 1$.
12. Lift the initial condition $u|_\Gamma = 0$ for the equation

    $$(Du)^2 = 1 \qquad (2)$$

    to the surface of the equation.

13. What is the geometric meaning of a solution of the (**??**) in $\mathbb{R}^n$ with the initial condition $u|_\gamma = 0$ for a smooth surface γ?

14. Find the maximal t_0 for which a solution of the problem $u_t + uu_x = \sin x$ is defined for $|t| < t_0,\ u|_{t=0} = 0$.

Solved in the class: 1, 2, 12.

HW: 3, 5 – 10.