Recitation and HW 6. D'Alambert method

1. Let $r = |x|, x \in \mathbb{R}^n,\ u(x) = f(r)$. Express Δu through f, f', f''.

2. Express Δu in the polar coordinates in the plane.

3. Deduce the d"Alambert formula.

   Here and below $\varphi_0 = -\sin x\chi_{[\pi,2\pi]}$.

4. Plot a movie for the Cauchy problem:

$$u_{t^2} = a^2 u_{x^2} \tag{1}$$

$$u|_{t=0} = \varphi_0,\ u_t|_{t=0} = 0. \tag{2}$$

5. The same for (1) and

$$u|_{t=0} = 0,\ u_t|_{t=0} = \varphi_0. \tag{3}$$

   In Problems 6 a – d the string is half-bounded: $x \geq 0$.

6. The same as in Problem 4 for $a.(1),(2), u|_{x=0} = 0$. $b.(1),(2), u_x|_{x=0} = 0$. $c.(1),(3), u|_{x=0} = 0$. $d.(1),(3), u_x|_{x=0} = 0$.

7. Solve the Cauchy problem for the bounded string $x \in [0, \pi]$

$$u_{t^2} = a^2 u_{x^2},\ u|_{x=0} = 0,\ u|_{x=\pi} = 0,\ u|_{t=0} = \sin kx,\ u_t|_{t=0} = 0.$$

8. Same for the initial data changed: $u|_{t=0} = 0,\ u_t|_{t=0} = \sin kx$.

9. Prove the parallelogram lemma:

$$(1) \Rightarrow u(A) + u(C) = u(B) + u(D),$$

   where A, B, C, D are vertexes of the characteritic parallelogram in the domain of u.

10. Solve the Cauchy problem for the bounded string $x \in [0, \pi]$

$$u_{t^2} = a^2 u_{x^2},\ u|_{x=0} = b(t),\ u|_{x=\pi} = c(t),\ u|_{t=0} = 0,\ u_t|_{t=0} = 0.$$

    Hint: the answer is an alternating sum of the values of b and c at the vertexes of characteristic broken lines.

11. Resonance. Consider the previous problem with $a = 1,\ b(t) = \sin kt$. Is the solution bounded?

Solved in the class: 3, 4, 5, 6a, 7, 9 .

HW: 1, 2, 6 b - d, 8, 10, 11