PDE-17, recitation 3, Linear and quasilinear partial differential equations

Linear equations

1. Find the general solution of the system
$$(x^2 - y^2)u_x + 2xyu_y = 0 \tag{1}$$
and plot the characteristics.

2. Are the following Cauchy problems for equation (1) solvable in a neighborhood of $(0,1)$:

   a) $u\,|_{x^2+y^2=1} = \sin x$

   b) $u\,|_{x^2+y^2=1} = \cos x$

3. a) Find the general solution of the problem $u_x + (-y + \cos 2x)u_y = 0$

   b) Is it correct that any closed simple curve on the (x,y) plane has a characteristic point?

   Quasilinear equations

4. Deduce an equation of a steady motion of particles in a no-collision media: $u(x,t)$ is the velocity of a particle that passes through the point x at the time t.

   Answer: $u_t + uu_x = 0$.

5. Solve the Cauchy problem $u_t + uu_x = 0$, $u|_{t=0} = f(x)$, problems a - c, and find the maximal segment $[a,b]$, $a \leq 0 \leq b$ for which the solution exists for any (t,x), $t \in [a,b]$, problems a - d:

   a) $f(x) = 1$;

   b) $f(x) = x$;

   c) $f(x) = -x$;

   d) $f(x) = \sin x$.

6. For what f from the previous problem the solution is defined on the whole (x,t) plane?

7. Solve the Cauchy problem $u_t - uu_x = 0$, $u|_{t=0} = f(x)$, and find the maximal segment $[a,b]$, $a \leq 0 \leq b$ for which the solution exists for any (t,x), $t \in [a,b]$:

   a) $f(x) = \alpha$;

   б) $f(x) = \alpha x$.

   Investigate the dependence of the answer on α.

8. Find the maximal segment $[a, b]$, $a \le 0 \le b$ for which the solution of the Cauchy problem $u_t + uu_x = 0$, $u|_{t=0} = f(x)$, exists for any (t, x), $t \in [a, b]$.

Solved in the class: 4, 5 a, b.

HW: 1, 2, 3a, b, 5d, 7, 8.