Recitation 7. Fourier method for the bounded string

1. Consider three functional spaces on the segment $[0, l]$:

$$u|_{x=0} = 0, \ u|_{x=l} = 0, \tag{1}$$

$$u_x|_{x=0} = 0, \ u|_{x=l} = 0, \tag{2}$$

$$u_x|_{x=0} = 0, \ u_x|_{x=l} = 0, \tag{3}$$

   Prove that the Laplace operator in any of these three spaces is self-adjoint.

2. Find the eigenvectors and eigenvalues of the Laplace operator in
   a. the space (1)
   b. the space (2)
   c. the space (3)

3. Find the Fourier series in the space (1) for $f(x)$, $l = \pi$:
   a. $f(x) = \text{sign} \left(x - \frac{\pi}{2} \right)$
   b. $f(x) = |x - \frac{\pi}{2}|$
   c. $f(x) = (x - \frac{\pi}{2})^2$

4. Solve the following mixed problems:

$$u_{tt} = u_{xx}, u|x = 0 = 0, \ u|x = \pi = 0, u|t = 0 = \varphi, \ u_t|_{t=0} = \psi$$

   a. $\varphi = |x - \frac{\pi}{2}|, \psi = 0$
   b. $\varphi = 0, \ \psi = (x - \frac{\pi}{2})^2.$

5. Solve the following mixed problems:
   a.
$$u_{tt} = u_{xx} + \sin x \sin t, u|_{x=0} = 0, \ u|_{x=\pi} = 0, u|_{t=0} = 0, \ u_t|_{t=0} = 0$$

   b.
$$u_{tt} = u_{xx}, u|_{x=0} = 0, \ u|_{x=\pi} = \sin \omega t, u|_{t=0} = 0, \ u_t|_{t=0} = 0.$$

   Investigate the answer with trespect to ω.

   Solved in the class: 1, 2a, 2b, 3a, 4a, 5a

   HW: 2c, 3b,c, 4b, 5b