Mid-semester examination. Program

April 24, 2017

1) Almost complex structures. Integrability. Beltrami equation.

2) Quasiconformal mappings. Lavrentiev–Morrey (Jr) Quasiconformal Mapping Theorem for the Riemann sphere. Non-holomorphic example of almost everywhere holomorphic sphere homeomorphism.

3) Basic complex dynamics. Montel Theorem (sketch of proof was given in a problem list). Fatou and Julia sets.

4) Basic examples and properties of Julia sets (non-emptyness, perfectness).

5) Classification of periodic components of the Fatou set.

6) Sullivan No Wandering Domain Theorem (with proof for simply connected wandering domain, with iterates being injective on it).

7) Theorem saying that each attracting periodic orbits attracts at least one critical orbit. Bound of the number of (super) attracting periodic orbits. Bound of the number of neutral periodic orbits (with a sketch-proof).

8) Hyperbolic rational functions. Fatou Conjecture. Mandelbrot set M: proof that $M \subset \overline{D}_2$.

9) J-stability: criterium and density. Holomorphic motions.

10) Structural stability: criterium and density.

11) Invariant line fields on the Julia set. No Invariant Line Fields Conjecture: relation to the Fatou Conjecture on density of hyperbolic functions.

12) Teichmüller spaces associated to rational functions.

13) Kleinian groups. Basic examples.

14) Basic properties of limit sets of Kleinian groups: density of fixed points; perfectness; common accumulation set of orbits in the discontinuity set.

15) Isometric action of automorphisms of the Riemann sphere on the hyperbolic 3-space. Ahlfors Measure Conjecture, now Calegari-Gabai-Agol theorem (statement). Sullivan's dictionary.

16) Fuchsian groups. Examples and main properties: $\Lambda \subset S^1 = \partial D_1$; either $\Lambda = S^1$, or Λ is nowhere dense in S^1.

17) Dirichlet fundamental domains. Properties. Criteria of finite generatedness and finiteness of type of quotient of Fuchsian group in terms of the Dirichlet domain.

18) Finiteness theorem for Fuchsian groups: criterium of type finiteness of quotient in terms of finite generatedness and the equality $\Lambda = S^1$. Proof using Dirichlet domains.

19) Ahlfors Finiteness Theorem (statement without proof). Proof of the particular case for the quotient of a simply connected component Ω_0 of the discontinuity set with finitely generated stabilizer.

20) Proof of the theorem saying that a torus with smooth almost complex structure is equivalent to a complex torus with the standard complex structure.

21) Proof of Poincaré–Köbe Uniformization Theorem using rectification of smooth almost complex structure on torus.

22) Grötzsch Inequality for quasiconformal annuli mappings.

23) Normality of the family of normalized K-quasiconformal homeomorphisms. Proof using Grötzsch Inequality.

24) Quasiconformal Mapping Theorem for measurable bounded almost complex structures on $\mathbb{C}$: proof of existence.

25) Quasiconformal Mapping Theorem for measurable bounded almost complex structures on $\mathbb{C}$: proof of uniqueness.

26) Analytic dependence of the rectifying quasiconformal mapping on parameter: statement without proof. Bers Simultaneous Uniformization Theorem.