Semester: Spring

Instructor: Alexander Esterov, Alexander Kolesnikov

Course description: This is an introductory course on variational methods, extremal problems, and convexity. In the first part of the course we discuss classical variational methods with examples of applications from geometry, physics, and engineering. We start with the variational calculus of Lagrange and Euler and finish with a brief discussion of the Pontryagin maximum principle. In the second (less standard) part we try to demonstrate the power of convex analysis and explain how convexity appears in various branches of science, starting with engineering and economical applications and finishing with abstract algebraic problems.

Prerequisites: standard linear algebra and analysis, ordinary differential equations. Some experience in functional analysis is desirable.

Curriculum:

- Classical extremal problems from geometry and physics (isoperimetric problem, brachistochrone, geodesics, and others).
- Euler-Lagrange equations.
- Pontryagin maximum principle.
- Applications of the Pontryagin maximum principle in physics and engineering.
- Convex functions, convex bodies, convex polytopes. Support functions, Minkowski summation.
- Legendre transform. Lagrangians and Hamiltonians.
- Hamilton–Jacobi equation. Bellman principle and Bellman equation.
- Mixed volumes, geometric inequalities (Brunn–Minkowski, Alexandrov–Fenchel).
- Minimax principle. Kuhn–Tucker theorem.
- Linear programming. Kantorovich duality.
- Elements of the game theory. Von Neuman theorem.
- Some applications in graph theory: bipatrite graphs, shortest path, tropical algebra.

Textbooks:

- Giaquinta M., Hildebrandt S., "Calculus of Variations." Vol. 1-2. Grundlehren der Mathematischen Wissenschaften, Berlin, Springer, 1996.
- Галеев Э.М., Зеликин М.И., Конягин С.В., "Оптимальное управление", МЦНМО, 2008.
- Gardner R.J., The Brunn-Minkowski inequality, Bull. Amer. Math. Soc. 39 (2002), 355–405
- Ziegler G.M., Lectures on Polytopes, Springer, Graduate Texts in Mathematics, Springer, 2007.

- Burago Yu.D., Zalgaller V.A., Geometric inequalities, Springer, 1988.
- Ferguson T.S., Linear programming: a concise intoduction.
- Ferguson T.S., Game theory. 2014.