

Affine Kac-Moody Lie algebras

Homework 1

due 04.10.17

0. Compute the dimension of the symplectic Lie algebra \mathfrak{sp}_{2n} .
1. Prove that all symmetric and wedge powers of the vector representation of \mathfrak{sl}_n are irreducible.
2. Let V be an irreducible representation of a Lie algebra. Prove that V^* is irreducible as well. For an irreducible representation V of \mathfrak{sp}_{2n} with highest weight λ find the highest weight of V^* .
3. Consider the following subalgebra inside \mathfrak{so}_4 :

$$\begin{pmatrix} 0 & a & 0 & 0 \\ -a & 0 & 0 & 0 \\ 0 & 0 & 0 & b \\ 0 & 0 & -b & 0 \end{pmatrix}.$$

Prove that this is a Cartan subalgebra and find the corresponding Cartan decomposition.

4. Let $R \subset \mathbb{R}^n$ be the set consisting of vectors of the form

$$R = \{\pm e_i, \pm 2e_i, 1 \leq i \leq n\} \cup \{\pm e_i \pm e_j, 1 \leq i, j \leq n, i \neq j\}$$

where e_i is the standard basis in \mathbb{R}^n . Prove that R is a (non reduced) root system (the BC_n system).

5. Compute the groups P/Q in types A_n and D_n , where P is the weight lattice and Q is the root lattice.
6. Decompose the tensor square of the adjoint representation of \mathfrak{sl}_3 into irreducible components.