GALOIS THEORY. HW2. (DUE OCTOBER, 27)

1. Prove that, for a positive integer N, an angle of measure $\frac{2\pi}{N}$ is trisectible (using a compass and a straightedge) if and only if 3 does not divide N.
2. Let $n_1, \cdots n_m$ be pairwise coprime integers such that, for every $1 \le i \le m$, $\sqrt[3]{n_i}$ is not an integer. Prove that

$$[\mathbb{Q}(\sqrt[3]{n_1} + \cdots + \sqrt[3]{n_m}), \mathbb{Q}] = 3^m.$$

3. Determine the Galois group of the splitting fields over $\mathbb{Q}$ of the following polynomials
(a) $x^4 + 2$
(b) $x^4 + x^2 + 1$
(c) $x^6 - 4$.
4. Let $F \subset K \subset L$ be algebraic extensions.
(a) Assume that $F \subset K$ and $K \subset L$ are normal. Is it true that $F \subset L$ is normal?
(b) Assume that $F \subset K$ and $K \subset L$ are separable. Is it true that $F \subset L$ is separable?
5. Let α and β be distinct roots of a polynomial $f(x) \in K[x]$ of degree $d > 1$. Prove that

$$[K(\alpha + \beta) : K] \le \frac{d(d-1)}{2}.$$

6. Is there a normal extension $\mathbb{Q} \subset F$ with $F \subset \mathbb{R}$ and $[F : \mathbb{Q}] = 3$?