# An introduction to cohomology theory: brief description and syllabus

A. Gorinov

gorinov@mccme.ru,agorinov@hse.ru

One of the main goals of algebraic topology is to answer the question whether two given topological spaces are homeomorphic or homotopy equivalent. This question and several related ones arise not only in topology, but also in mathematical physics, algebra and geometry of any kind. Classical cohomology and generalisations such as K-theory etc. is one of the main computational tools that in some cases allow one to answer this question.

The main prerequisites for this course are some basic algebra (groups, rings, fields), topology (topological and metric spaces, continuous maps, homotopy between continuous maps, coverings and the fundamental group) and category theory (categories, functors and natural transformations). We will recall some or all of them if necessary. This course is intended as a continuation of Topology 1 (HSE, 1st semester of 2017-2018; but also see below). Here is a tentative list of topics.

- Introduction. How to calculate the homology groups of surfaces.
- Singular homology. Basic homological algebra: exact sequences, complexes, 5-lemma, homotopy.
- (*) Homological algebra continued: acyclic models.
- First applications of acyclic models: homotopy invariance and excision for singular cohomology.
- CW-complexes, cellular homology and its particular cases and analogues. Simplicial complexes and simplicial homology. Smooth manifolds, Morse functions, handle decompositions and Morse homology.
- Homology and cohomology with coefficients. The universal coefficient theorems.
- The Künneth isomorphisms.
- Cup and cap products. Topological manifolds and the Poincaré duality.
- Lefschetz theorems. The contribution of a nondegenerate fixed point in the manifold case.
- Vector bundles and characteristic classes.
- Basic complex K-theory. Bott periodicity.
- An aside: generalised (co)homology. Spectra. Fibration and cofibration sequences.
- Applications of the complex K-theory: Hopf invariant 1, vector bundles and complex structures on spheres etc.

This list is quite long and we may not be able to cover all of these topics in equal detail in one semester. Instead we will focus on one of the following: (a) classical (co)homology theory; (b) characteristic classes; (c) K-theory, or a combination of these. Alternatively, we could cover generalised homology and stable homotopy theory in more detail.

The course will take place in the second semester of 2017-2018 and will be worth 5 credits. Time, venue and office hours: tba.

The main references are

- *Stable homotopy and generalised homology* by J.F. Adams.
- *A Course in Homotopy Theory* by D. Fuchs and A. Fomenko.
- *Algebraic Topology* by A. Hatcher, freely available online at `http://www.math.cornell.edu/~hatcher/AT/ATpage.html`.
- *Vector bundles and K-theory* by A. Hatcher, freely available online at `http://www.math.cornell.edu/~hatcher/VBKT/VBpage.html`.
- *Characteristic classes* by J. Milnor and J. Stasheff.

Occasionally we'll be using other sources as well.