

Topics in low-dimensional topology and algebraic geometry

The goal of low-dimensional topology is to understand manifolds of dimensions three and four, and this seminar will present a selection of techniques, new and old, which have been developed to achieve this. The course will begin from a topological perspective, but turning to recent developments in the subject (namely, those inspired by Floer homology) we will see that tools of algebraic geometry (in particular, character varieties) have a significant role to play.

In the topological sections, our emphasis will be on constructing examples and building geometric intuition. Note also that, regarding 3-manifold theory as consisting of two halves: knot theory and hyperbolic geometry, we will *not* touch on hyperbolic geometry; moreover, our approach to knot theory will be topological, *rather than* combinatorial.

Topics may include (in no particular order and not exclusively)

- Handlebody decompositions of manifolds, surgery and Morse theory
- Heegaard splittings and Heegaard diagrams of 3-manifolds
- Dehn surgery, and surgery on links in 3-space
- Representation spaces, character varieties and invariants from Heegaard diagrams
- Kirby calculus on 4-manifolds
- Special topics in 3-manifolds/knot theory (various invariants, branched coverings, the Poincaré conjecture, open book decompositions, foliations, fibered knots, etc.)
- Special topics in 4-manifolds (various invariants, exotic 4-manifolds, trisections of 4-manifolds, knotted surfaces, topological classification, etc.)

Organizers: Chris Brav (chris.i.brav@gmail.com, Rm 333) and Ash Lightfoot (alightfoot@hse.ru, Rm 309). These will play the roles of primary instructors, as well as advisors to students as they prepare for their presentation (see below).

Time: Every Thursday 12:00-13:30

Location: Room 208, Faculty of Mathematics of the Higher School of Economics, Usacheva str. 6 (ауд. 208 математического факультета ВШЭ по адресу ул. Усачева, 6.)

Prerequisites: All students are welcome. Though familiarity with differential topology (e.g., transversality) and basic algebraic topology (e.g., homology, CW-complexes, fundamental group) will certainly be helpful, students may use this as an opportunity to develop some intuition and motivation for concurrent or future courses in topology.

Assessment: Students will be assessed for attendance and are required to give a presentation at the seminar (or to one of the organizers) on material *appropriate to their level*. The course may be taken for **credit** through the Independent University of Moscow; see <http://ium.mccme.ru/f17/f17.html>

Selected reading:

- M. Abouzaid, C. Manolescu, *A sheaf-theoretic model for $SL(2, C)$ Floer homology*, <https://arxiv.org/abs/1708.00289>
- R. Gompf, A. Stipsicz, *4-manifolds and Kirby calculus*, American Math. Society, Providence (1999).
- P. Ozsváth and Z. Szabó, *Heegaard diagrams and holomorphic disks*, Different faces of geometry (2004) 301-348.
- D. Rolfsen, *Knots and Links*, Publish or Perish, Berkeley (1976)
- N. Saveliev, *Lectures on the Topology of 3-Manifolds*, Walter de Gruyter, Berlin (1999)