SCHUR-WEYL DUALITY VIA QUIVER HECKE ALGEBRAS

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ABSTRACT. Let J be a set of pairs consisting of good $U_q'(\mathfrak{g})$-modules and invertible elements in the base field $\mathbb{C}(q)$. The distribution of poles of normalized R-matrices yields Khovanov-Lauda-Rouquier algebras $R^J(\beta)$ for each $\beta \in Q^+$. We define a functor $\mathcal{F}_\beta$ from the category of graded $R^J(\beta)$-modules to the category of $U_q'(\mathfrak{g})$-modules. The functor $\mathcal{F} = \bigoplus_{\beta \in Q^+} \mathcal{F}_\beta$ sends convolution products of finite-dimensional graded $R^J(\beta)$-modules to tensor products of finite-dimensional $U_q'(\mathfrak{g})$-modules. It is exact if R^J is of finite type A, D, E. If $V(\varpi_1)$ is the fundamental representation of $U_q'(\widehat{\mathfrak{sl}}_N)$ of weight ϖ_1 and $J = \{(V(\varpi_1), q^{2i}) \mid i \in \mathbb{Z}\}$, then R^J is the Khovanov-Lauda-Rouquier algebra of type A_∞. The corresponding functor $\mathcal{F}$ sends a finite-dimensional graded R^J-module to a module in $\mathcal{C}_J$, where $\mathcal{C}_J$ is the category of finite-dimensional integrable $U_q'(\widehat{\mathfrak{sl}}_N)$-modules M such that every composition factor of M appears as a composition factor of a tensor product of modules of the form $V(\varpi_1)_{q^{2s}}$ $(s \in \mathbb{Z})$. Focusing on this case, we obtain an abelian rigid graded tensor category $\mathcal{T}_J$ by localizing the category of finite-dimensional graded R^J-modules. The functor $\mathcal{F}$ factors through $\mathcal{T}_J$. Moreover, the Grothendieck ring of the category $\mathcal{C}_J$ is isomorphic to the Grothendieck ring of $\mathcal{T}_J$ at $q = 1$.

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