Canonical Forms and Cauchy Problem
List 3 (6.02.2018)
Deadline — 6.03.2017.

1. Solve the Cauchy problem:

$$u_{xx} - u_{yy} - 2u_x - 2u_y = 8, \quad u|_{x=0} = -2y, \quad u_x|_{x=0} = 2(y-1).$$

2. What should be the function $g(x)$ for the solution of the Cauchy problem

$$u_{xx} + 5u_{xy} + 6u_{yy} = 0, \quad u|_{y=3x+2} = 4x^2 + 1, \quad u_x|_{y=3x+2} = g(x)$$

exists?

3. (a) Describe the oscillations of an infinite string occurring at $t \in (-\infty, +\infty)$ such that some interval of the string $(x_0 - \varepsilon, x_0 + \varepsilon)$ is static during all time of these oscillations.
   (b) The same question, but the interval of the string $(x_0 - \varepsilon, x_0 + \varepsilon)$ is static in $t \geqslant 0$.

4. Find the general solution of equation

$$u_{xx} + 2u_{xy} + 2u_{xz} + u_{yy} + 2u_{yz} + u_{zz} - u = 0.$$

5. Draw function graphs $u(x,t)|_{t=t_k}$, $t_k = \frac{kl}{4a}$, $k = 0, \ldots, 5$ where $u(x,t)$ is the Cauchy problem

$$u_{tt} = a^2 u_{xx}, \quad u|_{t=0} = u_0(x), \quad u_t|_{t=0} = v_0(x),$$

with initial data

(a) (*The guitar string.*) $v_0(x) \equiv 0$, $u_0(x) = \begin{cases} 0, & x \in (-\infty, l] \cup [3l, +\infty) \\ \dfrac{a}{l}x - a, & x \in [l, 2l] \\ -\dfrac{a}{l}x + 3a, & x \in [2l, 3l] \end{cases}$;

(b) (*The string of the piano.*) $u_0(x) \equiv 0$, $v_0(x) = \begin{cases} 0, & x \in (-\infty, l) \cup (2l, +\infty) \\ a, & x \in [l, 2l] \end{cases}$.