Task 1: formal and analytic normal forms, resonances

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Problem 1. Let $f(z) = \lambda z + O(z^2)$ be a germ of conformal mapping at 0, $0 < |\lambda| < 1$. Let U be a simply connected neighborhood of zero such that $f(\overline{U}) \Subset U$. Consider the *local orbit space* in $U \setminus \{0\}$: the space of forward orbits of the mapping $f|_{U\setminus\{0\}}$ (two orbits are *equivalent*, if they coincide up to a finite number of points). Prove that the local orbit space admits a natural structure of one-dimensional complex manifold that is conformally equivalent to a complex torus, and the orbit spaces corresponding to different U and to conformally conjugated germs are naturally isomorphic. Find the modulus of the above complex torus as a function of λ.

Definition 1. Consider a mapping $z \mapsto f(z)$, $z = (z_1, \dots, z_n)$, $f = (f_1, \dots, f_n)$. We say that the mapping f is *triangular,* if $f_j = f_j(z_j, \dots, z_n)$.

Problem 2. Let $f(z) = \Lambda z + O(||z||^2)$ be a germ of n-dimensional holomorphic mapping at the origin.

a) Prove that if $\Lambda = f'(0)$ has *distinct* eigenvalues $\lambda_1, \dots, \lambda_n \in \mathbb{C}$ with $0 < |\lambda_1| \leq \dots \leq |\lambda_n| < 1$, then each vector function consisting only of resonant monomials is triangular.

b) Deduce that if a n-dimensional vector polynomial in n variables with non-zero derivative at zero consists only of resonant monomials, then the inverse of its germ at 0 is also a vector polynomial.

c) In the latter case find out whether all the monomials of the inverse vector polynomial are always resonant.

Problem 3. In the above problem find all the resonant vector monomial for

a) $\Lambda = \mathrm{diag}(1, 2, 4)$;

b) $n = 1$, $\Lambda = 0$;

c) $n = 2$, $\Lambda = \mathrm{diag}(\frac{1}{2}, \frac{1}{8})$.

Problem 4. State and solve problems similar to Problems 1 and 2 for germs of holomorphic vector fields.