Partial differential equations
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1. Characteristics method for second order PDE on two variables.
2. Characteristics method (multi-variables case).
3. Cauchy problem for string equation. Existence and uniqueness.
4. First order partial differential equations, general theory. Cauchy problem: existence and uniqueness:
- linear case;
- general case (contact structures etc).

5. Heat equation (in any dimension), Cauchy problem, existence and uniqueness.

6. Laplace equation and harmonic functions. Maximum Principle. The Mean Value Theorem for harmonic functions and its inverse for C^2-smooth functions: any C^2-smooth function satisfying the Mean Value Equality for every sphere is harmonic.

7. Harmonic functions in two variables as real parts of holomorphic functions. Conformal invariance of harmonicity. Expression of Laplace operator via dz, $\overline{dz}$.

8. Dirichlet Problem for Laplace equation in a bounded domain. Uniqueness of solution. Deduction of the formula for its solution in a two-dimensional disk via conformal automorphisms of the unit disc and Mean Value Theorem.

9. Dirichlet Problem for Laplace equation in any dimension. Green function, its properties (symmetry, harmonicity in both variables,...) Construction of the Green function for a ball. Deduction of Poisson formula for solution of the Dirichlet Problem in a ball.

10. Dirichlet Problem in arbitrary bounded domain with piecewise smooth boundary. Sobolev spaces. Friedrichs Inequality (equivalence of two norms in H_0^1). Distributions. Dirichlet problem in the sence of distributions: existence and uniqueness of solution.

11. Weyl's Lemma: each distribution that is harmonic in the sense of distributions is a true harmonic function.

12. Wave equation in higher dimensions. Conservation Law for "symplectic structure": for any two solutions $u(t,x)$, $v(t,x)$ that are differentiable mappings from t to the Schwarz space of functions $f(x)$ one has

$$\int_{\mathbb{R}^n} (\dot{u}v - \dot{v}u)dx \equiv const, \ \dot{f} := \frac{\partial f}{\partial t}.$$

Cauchy problem in three dimensions. Spherical waves (solutions depending only on t and $|x|$: general form. Deduction of Kirchoff Formula for its solution assuming that we are dealing with solutions in Schwarz space in x and using the Conservation Law.

13. Proof of uniqueness of solution of Cauchy problem for wave equation in three dimensions.

14. Proof that the Kirchoff Formula gives a solution of the Cauchy problem.

15. Cauchy problem for wave equation in two dimensions. Descent method: deduction of Poisson Formula for solution in two dimensions from Kirchoff Formula in three dimensions.

16. Petrovsky correct equations. Formula for solution via Fourier transform. Petrovsky correctness of the Heat and Schrödinger equations. Deduction of formula for solution of Schrödinger equation.

17. Cauchy–Kovalevskaya theorem. Hadamard's example.