LATTICE POLYTOPES OF SMALL VOLUME
REU project at NRU HSE

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Modern geometry discovered a number of important relations between algebraic geometry and geometry of polytopes, see below for an example. Many times these relations have already allowed to solve an open problem in one of these fields with the techniques from the other one. Our project is devoted to a question about lattice polytopes, motivated by algebraic geometry.

A lattice polytope is a polytope with integer coordinates of the vertices. It turns out that there are finitely many non-isomorphic lattice polytopes of a given volume (polytopes are said to be isomrophic if they can be identified by a linear map of their ambient spaces). In particular, the list of all isomorphism classes of polytopes of volume at most 4 is known.

This classification is motivated by a relation to algebraic geometry: the notion of the Newton polytope establishes a one-to-one correspondence between n-dimensional lattice polytopes of volume V and general systems of n polynomial equations with V solutions. In particular, such system is solvable by radicals iff V is at most 4 (generalizing the celebrated Abel--Ruffini insolvability theorem for one univariate polynomial equation). Thus, the aforementioned classification of polytopes essentially classified general systems of equations solvable by radicals.

In our project, we shall try to obtain further results in this direction: classify lattice polytopes of higher volume, tuples of polytopes of a given small mixed volume, or to estimate how fast this classification grows as we increase the volume. For instance, here is the conjectural classification of triples of lattice polytopes of mixed volume 2: