Exam program (final version)

During the exam you *are not* allowed to use any notes, mobile phones, neighbours, etc. Of course, you should prove only those statements which were proven during the lectures or which were formulated as exercises (see the list of exercises). Pay special attention to the main objects and concepts, e.g. stationary measure, ergodicity, etc., and to the related assertions and their proofs.

1. Markov chains with finite number of states: the probability model.
2. Transition probability matrix. Stochastic matrices. Iterations of a Markov chain: formula for the probability distribution at the n-th step. Homogeneous Markov chains. The Kolmogorov-Chapman equation.
3. Examples of Markov chains: random walks, the Galton-Watson birth-death process, the example from genetics, the Ehrenfest model.
4. Stationary states of Markov Chains. Their existence: the Bogoliubov-Krylov method.
5. Ergodic theorem for Markov chains (with ergodic transition probability matrices).
6. The law of large numbers for Markov chains.
7. On the ergodic theorem for Markov chains from the linear algebra of the transition probability matrix (several lemmas on this subject which were proven during the lecture).
8. The Perron-Frobenius theorem.
9. Topological structure of Markov chains: the relations "leads to", "communicates with" and their properties. Equivalence classes, partial ordering and minimal equivalence classes.
10. Essential and inessential states. Exponential decay of probability to stay in the set of inessential states. Absorbing states.
11. Periodic Markov chains: period of a state, period is a class property.
12. Aperiodic Markov chains. Ergodic theorem for irreducible aperiodic Markov chains.